Numerical Infinities for Mathematics Education

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In this talk I reflect on the impact of Yaroslav Sergeyev's computational treatment of infinitely large and small numbers on mathematics education, with reference to recent interventions, their results, and future suggestions.

My starting point is a simple observation. The high school curriculum, as well as pre-university or early university courses, include many basic topics (e.g. arithmetical and geometric progressions, sequences and series, basic probability) that train students to acquire familiarity with certain elementary forms of computation or modelling. Students are then prepared to select and use familiar techniques in more demanding contexts, but they are typically not prepared to think like a mathematician, i.e. to engage not only in the application, but also in the construction of methods.

An interesting and important question for mathematics education is therefore the following: is there a way of reconstructing basic mathematical training that does not only provide basic competence but also contributes to the genuine development of mathematical creativity?

I propose a positive answer by looking at recent work in schools that has employed the computational methodology proposed by Prof. Sergeyev (see [6] for a survey), especially [1, 2, 3]. This methodology does not only naturally apply to all of the basic topics mentioned earlier, but provides for each of them an alternative outlook, which paves the way towards asking new questions and solving new problems. As shown in [4], students are sensitive to this opportunity and can independently make use of it. I rely on examples of tasks and worksheets from [5] to illustrate interesting contexts in which it is possible very naturally to motivate students to engage in autonomous discovery.

References

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