

BOOK OF ABSTRACTS

The 5th International Conference
and Summer School

Numerical Computations: Theory and Algorithms NUMTA 2026



edited by
Yaroslav D. Sergeyev
Dmitri E. Kvasov
Annabella Astorino



UNIVERSITÀ DELLA CALABRIA

DIPARTIMENTO DI
INGEGNERIA INFORMATICA,
MODELLISTICA, ELETTRONICA
E SISTEMISTICA

DIMES

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of the 5th International Conference and Summer School

Numerical Computations: Theory and Algorithms

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Department of Computer Engineering, Modelling, Electronics and Systems Science

University of Calabria

in cooperation with the Society for Industrial and Applied Mathematics (SIAM), USA

Falkensteiner Club Funimation Garden Calabria

Curinga (CZ), Italy

June 21–27, 2026

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Dipartimento di Ingegneria Informatica, Modellistica, Elettronica e Sistemistica
Università della Calabria

Edizione eBook 2026

ISBN: 978-88-907488-4-4

Via Pietro Bucci 42C - 87036 Rende (CS) - Italy

Tel. (+39) 0984 494718

Sito web : <https://dimes.unical.it>

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NUMERICAL COMPUTATIONS: THEORY AND ALGORITHMS

Fifth International Conference and Summer School NUMTA–2026

21–27 June 2026, Falkensteiner Garden Calabria, Italy

Dear Participants,

Welcome to the Fifth International Conference and Summer School NUMTA–2026 “Numerical Computations: Theory and Algorithms”. The Conference is organized by the Department of Computer Engineering, Modeling, Electronics and Systems Science of the University of Calabria, Italy. We are proud to inform you that, as usual, NUMTA–2026 is organized in cooperation with the Society for Industrial and Applied Mathematics (SIAM), USA. The previous editions of NUMTA took place in several beautiful places in Calabria in 2013, 2016, 2019, and 2023.

The goal of all NUMTA Conferences is to create a multidisciplinary round table for an open discussion on the nature of numerical modeling by using traditional and emerging computational paradigms. The NUMTA-2026 Conference will discuss all aspects of numerical computations and modeling from foundations, philosophy and teaching to advanced numerical techniques. New technological challenges and fundamental ideas from theoretical computer science, machine learning, linguistic, logic, set theory, and philosophy will meet requirements and new fresh applications from engineering, physics, chemistry, biology, economy, and teaching mathematics.

Researchers from both theoretical and applied sciences have been invited to use this excellent possibility to exchange ideas with leading scientists from different research fields. Papers discussing new computational paradigms, relations with foundations of mathematics, and their impact on natural sciences have been particularly solicited. A special attention during the Conference will be dedicated to numerical optimization techniques and a variety of issues related to theory and practice of the usage of infinities and infinitesimals in numerical computations. In particular, there will be a substantial number of talks dedicated to a recent promising methodology allowing one to execute numerical computations with finite, infinite, and infinitesimal numbers on a new type of a supercomputer – the Infinity Computer patented in several countries.

We are happy to inform you that researchers from the following 44 countries have submitted their contributions to the Conference: Algeria, Argentina, Australia, Austria, Brazil, Canada, China, Czechia, Denmark, Estonia, Finland, France, Germany, Greece, India, Ireland, Israel, Italy, Japan, Kazakhstan, Kenya, Kuwait, Latvia, Lebanon, Lithuania, Malaysia, Mexico, Morocco, New Zealand, Philippines, Poland, Portugal, Russia, Saudi Arabia, Serbia, Singapore, Slovakia, Spain, Switzerland, Turkey, Uganda, United Kingdom, United States, Vietnam.

The abstracts in this volume are arranged alphabetically according to the first author. For abstracts with multiple authors, the author registered as the conference presenter is underlined. An author index is also provided, with hyperlinks that lead directly to the corresponding abstracts.

Numerous plenary lectures and tutorials will be delivered by scientists who are undiscussed leaders in their research fields. As is customary at NUMTA conferences,

an award will be presented in recognition of outstanding achievements in the scientific fields covered by NUMTA–2026. Special recognition will also be given to the best presentation delivered by a young scientist.

Post conference Proceedings of NUMTA–2026 will be published by Springer as a dedicated volume of the prestigious collection “Lecture Notes in Computer Science”. Authors of selected talks will be invited to submit full papers to special issues of the premier international journals dedicated to the Conference.

We wish all participants productive and successful work during the Conference and hope that the Conference will give you a lot of inspiration leading to new important results in your research fields.

Yaroslav D. Sergeyev
Chairman of NUMTA–2026
Distinguished Professor
Head of Numerical Calculus Laboratory
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Plenary lectures and tutorials

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Infinite Update Networks on Finite Graphs using Grossone

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Keywords. Infinite games; networks; grossone.

Communication networks can be modeled and their problems analyzed using the concept of infinite games on finite graphs. For example, in a distributed database system a problem of redundancy of data can be resolved by sharing information between all nodes of the system. This can be accomplished by a packet of current data continuously running through all nodes of the distributed system. Hence this is a solution without termination (an infinite update) and was originally shown by McNaughton (see [3] and [4]). In [1], the theory of grossone (see [2] and [5]) and the infinite unit axiom have been applied to develop a finer and more precise theory of infinite games on finite graphs. In this paper, the work on infinite games is extended to update networks, whereby each network node is updated infinitely often. By using grossone, the number of visitations to each node can be counted with precision so that each node is visited the same number of times and hence creates a “balanced” update network. Here update networks are studied, by using graph theory arguments, and a decision process is built to determine when it is possible to create a balanced network with infinite node visitations.

References

- [1] D’Alotto L. (2020) Infinite games on finite graphs using grossone. *Soft Computing*, vol. 24, pp. 17509–17515.
- [2] Lolli G. (2025) Infinite numbers, infinity computing: The philosophy of grossone. *Soft Computing*, vol. 29, pp. 4287–4299.
- [3] McNaughton R. (1993) Infinite games played on finite graphs. *Annals of Pure and Applied Logic*, vol. 65, pp. 149–184.
- [4] Khoussainov B., Nerode A. (2001) *Automata Theory and its Applications*, Birkhäuser Boston, USA.
- [5] Sergeyev Y.D. (2017) Numerical infinities and infinitesimals: Methodology, applications, and repercussions on two Hilbert problems. *EMS Surv. Math. Sciences*, vol. 4, pp. 219–320.

Nonlinear Optimization, Machine Learning and Grossone

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Keywords. Nonlinear optimization; machine learning; grossone.

The introduction of the grossone methodology, developed by Sergeyev, provides a novel computational framework to deal numerically with infinite and infinitesimal quantities within a unified positional numeral system based on the infinite unit ① (grossone) [4]–[6]. This approach has been successfully applied to optimization theory, allowing the construction of exact differentiable penalty functions for constrained nonlinear programs [3]. Penalty parameters traditionally required to tend to $+\infty$ can be replaced by suitable powers of grossone, and the finite part of the resulting solution recovers the optimal solution of the original problem, while infinitesimal terms encode Lagrange multipliers. Nonlinear optimization plays a central role in modern Machine Learning (ML), where most learning tasks are formulated as the minimization of empirical risk functionals possibly enriched with regularization terms. Here, grossone has been employed to approximate important problems such as the ℓ_0 pseudo-norm through smooth expressions involving infinitesimals [2], the construction of spherical, linear and conical separators [7, 8]. The interplay between nonlinear optimization, ML models, and the grossone methodology opens promising perspectives by providing exact penalty constructions, smooth approximations of combinatorial terms, and a unified numerical treatment of infinite parameters. Grossone-based techniques may therefore become a powerful tool for large-scale learning problems where sparsity, constraints, and nonlinear structures coexist.

References

- [1] Shara J. (2022) Solving a ML problem using the grossone. *International Journal of Current Science Research and Review*, vol. 5(03), 856–862.
- [2] De Leone R., Egidi N., Fatone L. (2020) The use of grossone in elastic net regularization and sparse support vector machines. *Soft Computing*, vol. 24(23), 17669–17677.
- [3] De Leone R. (2022) The role of grossone in nonlinear programming and exact penalty methods. In: Sergeyev Y.D., De Leone R. (eds.), *Numerical Infinities and Infinitesimals in Optimization*, ECC, vol. 43, pp. 249–269. Springer, Cham.
- [4] Sergeyev Y.D. (2003) *Arithmetic of Infinity*, Edizioni Orizzonti Meridionali.
- [5] Sergeyev Y.D. (2008) A new applied approach for executing computations with infinite and infinitesimal quantities. *Informatica*, vol. 19(4), 567–596.
- [6] Sergeyev Y.D., De Leone R. (Eds.) (2022) *Numerical Infinities and Infinitesimals in Optimization*, Emergence, Complexity and Computation, vol. 43. Springer, Cham.
- [7] Astorino A., Fuduli A. (2022) Comparing linear and spherical separation using grossone-based numerical infinities in classification problems. In: Sergeyev Y.D., De Leone R. (eds.), *Numerical Infinities and Infinitesimals in Optimization*, ECC, vol. 43, pp. 249–269. Springer, Cham.
- [8] Astorino A., Fuduli A., Gaudio M. (2010) DC models for spherical separation. *Journal of Global Optimization*, vol. 48(4), 657–669.

Active Inference for Distributed Intelligence in the Computing Continuum

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Modern distributed systems deal with uncertain scenarios, where environments, infrastructures, and applications are widely diverse. In the scope of IoT-Edge-Fog-Cloud computing, leveraging neuroscience-inspired principles and mechanisms could aid in building more flexible solutions able to generalize over different environments. A captivating set of hypotheses from the field of neuroscience suggests that human and animal brain mechanisms result from a few powerful principles. If proved to be accurate, these assumptions could open a deep understanding of the way humans and animals manage to cope with the unpredictability of events and imagination. In this talk, we will explore how Active Inference mechanisms can be utilized for Distributed Intelligence in the Computing Continuum. This has considerable impact for all research related to IoT and AI in general.

References

- [1] Dustdar S. et al. (2023) On distributed computing continuum systems. *IEEE Transactions on Knowledge and Data Engineering*, vol. 35, no. 4, pp. 4092–4105.
- [2] Sedlak B. et al. (2024) Equilibrium in the computing continuum through active inference. *Future Generation Computer Systems*, vol. 160, pp. 92–108.
- [3] Pujol V.C. et al. (2024) On causality in distributed continuum systems. *IEEE Internet Computing*, vol. 28, no. 2, pp. 57–64.
- [4] Pujol V.C. et al. (2025) Distributed intelligence in the computing continuum with active inference. [arXiv:2505.24618](https://arxiv.org/abs/2505.24618), <https://doi.org/10.48550/arXiv.2505.24618>

From System Modeling to System Simulation: Systemic Digital Twins for Improving Industrial Operations

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The talk will discuss the fundamentals of Σ , a formal system specification language with a sound mathematical basis that allows to specify the behavior and the structure of an industrial system and to automatically compile a new type of simulation tool—the systemic digital twin—from that specification. We will illustrate the power and scope of Σ on various industrial case studies.

Towards Optimal Offline Reinforcement Learning

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We study offline reinforcement learning problems with a long-run average reward objective. The state-action pairs generated by any fixed behavioral policy thus follow a Markov chain, and the empirical state-action-next-state distribution satisfies a large deviations principle. We use the rate function of this large deviations principle to construct an uncertainty set for the unknown true state-action-next-state distribution. We also construct a distribution shift transformation that maps any distribution in this uncertainty set to a state-action-next-state distribution of the Markov chain generated by a fixed evaluation policy, which differs from the unknown behavioral policy. We prove that the worst-case average reward of the evaluation policy with respect to all distributions in the shifted uncertainty set provides the least conservative estimator for the average reward under the unknown true distribution. The emerging robust optimization problem can be viewed as a robust Markov decision process with a non-rectangular uncertainty set, and we develop an efficient policy gradient algorithm to solve this problem.

Next Generation AI Computing

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Artificial intelligence is driving breakthrough after breakthrough across industry, public life, and the sciences. Yet, despite these achievements, today's AI systems face two critical challenges: limited reliability and immense energy demands.

This lecture approaches these issues from a computational perspective. We will examine the fundamental constraints of current AI technology, including problems of computability, the energy inefficiency of digital hardware, and the phenomenon of complexity blowup when solving PDEs. The discussion will then shift to emerging computing paradigms such as neuromorphic and analog computing, which promise a radical rethinking of how AI is built and operated. These approaches could pave the way toward AI systems that are not only vastly more energy efficient, but also inherently more robust, reliable, and trustworthy.

Asymmetric Long-Step Primal-Dual Interior-Point Methods with Dual Centering

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We discuss a new way of development of asymmetric Interior-Point Methods for solving primal-dual problems of Conic Optimization. It is very efficient for problems, where the dual problem is simpler than the primal one. The problems of this type arise often in Semidefinite Optimization (SDO), for which we propose a new primal-dual method with very attractive computational cost.

In this approach, we do not need sophisticated Linear Algebra, restricting ourselves by standard Cholesky factorization. However, our complexity bounds correspond to the best-known polynomial-time results. Moreover, for symmetric cones the bounds automatically depend on the minimal barrier parameter between the primal and the dual feasible sets. We show by SDO-examples that the corresponding gain can be very big. We discuss some classes of SDO-problems, where the complexity bounds are proportional to the square root of the number of linear equality constraints and the computational cost of one iteration is as in Linear Optimization. Our theoretical developments are supported by encouraging numerical testing.

Impact of AI and Optimization on the Economics of Sustainability

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The transformative impact of AI on the economics of sustainability lies in its ability to optimize resource use, reduce waste, and drive innovation, fundamentally aligning economic systems with sustainable practices. By leveraging predictive analytics and intelligent automation, AI enhances renewable energy integration, streamlines agriculture, and advances circular economy models, such as precision recycling. It also accelerates the adoption of green technologies, like electric vehicles and smart cities, enabling governments and businesses to achieve sustainability goals while fostering economic growth. Ultimately, AI harmonizes economic progress with environmental stewardship, positioning itself as a key enabler of sustainable and resilient economies.

Numerical Infinities for Mathematics Education

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Keywords. Mathematics education; infinitely large and small numbers; basic probability.

In this talk I reflect on the impact of Yaroslav Sergeyev’s computational treatment of infinitely large and small numbers on mathematics education, with reference to recent interventions, their results, and future suggestions.

My starting point is a simple observation. The high school curriculum, as well as pre-university or early university courses, include many basic topics (e.g. arithmetical and geometric progressions, sequences and series, basic probability) that train students to acquire familiarity with certain elementary forms of computation or modelling. Students are then prepared to select and use familiar techniques in more demanding contexts, but they are typically not prepared to think like a mathematician, i.e. to engage not only in the application, but also in the construction of methods.

An interesting and important question for mathematics education is therefore the following: is there a way of reconstructing basic mathematical training that does not only provide basic competence but also contributes to the genuine development of mathematical creativity?

I propose a positive answer by looking at recent work in schools that has employed the computational methodology proposed by Prof. Sergeyev (see [6] for a survey), especially [1–3]. This methodology does not only naturally apply to all of the basic topics mentioned earlier, but provides for each of them an alternative outlook, which paves the way towards asking new questions and solving new problems. As shown in [4], students are sensitive to this opportunity and can independently make use of it. I rely on examples of tasks and worksheets from [5] to illustrate interesting contexts in which it is possible very naturally to motivate students to engage in autonomous discovery.

References

- [1] Ingarozza F. et al. (2020) A grossone-based numerical model for computations with infinity: A case study in an Italian high school. *LNCS* 11973, pp. 329–342.
- [2] Nasr L. (2023) Students’ resolutions of some paradoxes of infinity in the lens of the grossone methodology. *Informatics and Education*, vol. 38, pp. 83–91.
- [3] Nasr L. (2022) The effect of Arithmetic of Infinity methodology on students’ beliefs of infinity. *Mediterranean J. for Research in Math. Education*, vol. 19, pp. 5–19.
- [4] Iannone P. et al. (2018) Investigating secondary school students’ epistemologies through a class activity concerning infinity. In: Proc. of the 42nd Conference of the International Group for the Psychology of Mathematics Education (E. Bergqvist et al., eds.). Vol. 3, pp. 131–138. Umeå, Sweden: PME.
- [5] Rizza D. (2023) *Primi Passi nell’Aritmetica dell’Infinito*. Bonomo Ed., Bologna.
- [6] Sergeyev Y.D. (2017) Numerical infinities and infinitesimals: Methodology, applications, and repercussions on two Hilbert problems. *EMS Surveys in Mathematical Sciences*, Vol. 4, pp. 219–320.

Grossone Infinity Computing: Foundations and Computations with Floating-Point Infinities and Infinitesimals

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Keywords. Infinity Computing; grossone; infinities and infinitesimals.

In this tutorial – designed for a broad audience and requiring no special mathematical background – we present a recent computational methodology (see [2–4]) that makes it possible to work with infinities and infinitesimals numerically within a unified computational framework. This is fundamentally different from traditional approaches, which treat infinite and infinitesimal quantities only symbolically and rely on different notions of infinity in different contexts admitting numerous paradoxes (see [1]).

The methodology is proposed to be implemented on a new type of supercomputer, the *Infinity Computer*, which employs a patented floating-point representation with the infinite radix ①, called *grossone*. One of its major strengths lies in its computational effectiveness in practical applications, as extensively discussed at NUMTA conferences by numerous colleagues (see, e.g., [5]).

Through a series of engaging examples, we show that the ①-based methodology is useful from both computational and theoretical perspectives. In particular, we demonstrate that several classical paradoxes involving infinity and infinitesimals can be avoided within this framework (see [4]).

A lot of additional information can be found at the following web pages:

<https://www.theinfinitycomputer.com> & <http://www.numericalinfinities.com>.

References

- [1] Cantor G. (1955) *Contributions to the founding of the theory of transfinite numbers*. Dover Publications, New York.
- [2] Sergeyev Y. D. (2003, 2nd ed. 2013) *Arithmetic of Infinity*. Orizzonti Meridionali, Cosenza.
- [3] Sergeyev Y. D. (2017) Numerical infinities and infinitesimals: Methodology, applications, and repercussions on two Hilbert problems. *EMS Surveys in Mathematical Sciences*, vol. 4(2), 219–320.
- [4] Sergeyev Y. D. (2022) Some paradoxes of infinity revisited. *Mediterranean Journal of Mathematics*, vol. 19(3), article 143.
- [5] Sergeyev Y. D., De Leone R. (eds.) (2022) *Numerical Infinities and Infinitesimals in Optimization*. Springer, Cham.

Presentations from special sessions and regular presentations

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Statistically Validated Lasso for Gaussian Graphical Models in Financial Networks

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Keywords. Graphical model; elastic net penalty; statistically validated network.

In this paper we present IsLasso-SVN, a novel two-stage procedure for estimating sparse Gaussian graphical models in financial networks. The method first estimates the precision matrix through induced-smoothed lasso regressions, which retain lasso-type sparsity while also providing standard errors and p -values for the regression coefficients. In a second step, the resulting conditional dependence network is filtered through a statistically validated network procedure based on hypergeometric testing and multiple-comparison correction, so that only links unlikely to arise from random connectivity are retained.

We evaluate the method on simulated network structures and compare its performance with existing alternatives under different sample sizes. We further apply the procedure to daily S&P 500 sector log-return residuals over the period 2018-2020, uncovering how the conditional dependence structure among U.S. economic sectors shifts markedly around the onset of the COVID-19 pandemic.

Acknowledgements. This study was partly funded by the NextGenerationEU initiative in the framework of the Italian GRINS (Growing Resilient, Inclusive and Sustainable) project (project GRINS PE00000018 - CUP B73C22001260006).

References

- [1] Friedman J., Hastie T., Tibshirani R. (2007) Sparse inverse covariance estimation with the graphical lasso. *Biostatistics*, vol. 9(3), pp. 432–441. DOI: 10.1093/biostatistics/kxm045.
- [2] Cilluffo G., Sottile G., La Grutta S., Muggeo V.M.R. (2020) The induced smoothed lasso: A practical framework for hypothesis testing in high dimensional regression. *Statistical Methods in Medical Research*, vol. 29(3), pp. 765–777. DOI: 10.1177/0962280219842890.
- [3] Tumminello M., Micciche S., Lillo F., Piilo J., Mantegna R.N. (2011) Statistically validated networks in bipartite complex systems. *PLOS ONE*, vol 6(3), pp.1–11. DOI: 10.1371/journal.pone.0017994.
- [4] Bernardini D., Paterlini S., Taufer E. (2025) New estimation approaches for graphical models with elastic net penalty. *Econometrics and Statistics*, vol. 33, pp.258–281. DOI: 10.1016/j.ecosta.2022.06.003.

A Parallel Two-Stage Vector-Based Hybrid Genetic Algorithm for Solving a Large-Scale Fixed Capital Formation Model

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Keywords. Optimal control; fixed capital formation; parallel genetic algorithms; differential evolution.

The search for optimal solutions in contemporary models of fixed capital formation presents a complex optimization problem. It is necessary to determine the most preferable moments for both fixed capital investments and disposals in order to achieve optimal growth within a multi-regional multi-sectoral system [1]. Therefore, a new parallel two-stage vector-based hybrid genetic algorithm (VBHGA) has been developed. The VBHGA uses similar approaches to crossover operation as the earlier suggested MBHGA algorithm [2], based on mutual exchanging segments of decision variable matrices (vectors) between parent solutions to form offspring individuals. Such decisions determine the entire set of optimal moments for fixed capital investment and disposal to maximize the output for each region and sector using principles of integral or differential optimization. Unlike other parallel genetic algorithms that use matrix-based crossover during the entire evolutionary search process, the VBHGA algorithm uses differential evolution (DE) at the first stage to massively generate an initial population of binary decisions using real number vectors, which are the parameters of heavy-tailed distributions. As a result, the initial population is closer to the global optimum than that formed randomly. In the second stage, a binary-coded genetic algorithm (GA) with GPU acceleration is applied during the entire evolutionary search process, using the vector crossover operator. The developed algorithm provides higher time efficiency and accuracy of the solutions obtained compared to other well-known evolutionary algorithms when solving large-scale models of fixed capital formation.

References

- [1] Calvo G. A. (1976) Optimal Growth in a Putty-Clay Model. *Econometrica*, 44(5), pp. 867–878.
- [2] Akopov A. S. (2025) MBHGA: A Matrix-Based Hybrid Genetic Algorithm for Solving an Agent-Based Model of Controlled Trade Interactions. *IEEE Access*, vol. 13, pp. 26843–26863.

Entropy-Guided Least-Squares Bivariate Splines for Robust Smoothing of Noisy Data

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Keywords. Robust regression; outliers detection; entropy; bivariate splines.

We propose a weighted least-squares approximation framework in which both the approximating function and the error weights are determined simultaneously through a maximum-entropy principle. Interpreting the weights as a probability distribution, they are selected by maximizing the entropy—i.e., the degree of uncertainty—under the constraint that the mean squared error remains below a prescribed threshold. This constraint acts as a regularization mechanism, yielding a robust regression method capable of automatically identifying and down-weighting outliers during the fitting process.

The resulting strategy is incorporated into a bivariate spline regression model and tested on several illustrative examples, including noisy point clouds and real-world image processing problems.

Moreover, motivated by recent theoretical results on existence and uniqueness obtained by reformulating the problem as an ODE-IVP, we analyze the sparsity structure of the associated Jacobian matrix in order to design suitable integration strategies.

Acknowledgements.

This research was supported by INdAM-GNCS Project, Italy, and by the National Recovery and Resilience Plan, Mission 4 Component 2 - Investment 1.3 - Future Artificial Intelligence Research - FAIR - PE00000013 - Spoke 6 - Symbiotic AI, under the NRRP MUR program funded by the European Union - NextGenerationEU.

References

- [1] Amodio P., Brugnano L., Iavernaro F. (2026) On the use of the principle of maximum entropy to improve the robustness of bivariate splines least-squares approximation. *Mathematics and Computers in Simulation*, vol. 260, pp. 698–712.
- [2] Brugnano L., Giordano D., Iavernaro F., Rubino G. (2024) An entropy-based approach for robust least squares spline approximation. *J. Comput. Appl. Math.*, vol. 443, article 115773.
- [3] Giordano D., Iavernaro F. (2021) Maximal-entropy driven determination of weights in least-square approximation. *Math. Meth. Appl. Sci.*, vol. 44, pp. 6448–6461.

Conics and Grossone: Algebraic Continuity in the Ellipse-Parabola Transition

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Keywords. Conic sections; focal points; Grossone.

The classification of conic sections is a cornerstone of geometry, yet the transition from an ellipse to a parabola remains a problematic singularity in the classical real number system $\mathbb{R}$. In the standard framework, as the eccentricity approaches unity, the major axis diverges to infinity and the secondary focus is lost to a limit process, resulting in a topological rupture between closed and open curves. This presentation proposes a novel resolution to this discontinuity by leveraging Sergeyev’s numerical system.

By redefining the parabola as an “infinitely long ellipse” whose parameters are expressed through infinite and infinitesimal numerals, we demonstrate that it is possible to maintain the curve’s focal integrity and algebraic closure even at infinite scales. We provide exact expressions for the secondary focus position and the total area of these structures—quantities that are inherently undefined in classical calculus. This approach offers a unified perspective on conics and provides a robust mathematical foundation for addressing singularities in applied domains, such as geometric optics. By bridging the gap between finite observations and infinite models, this framework preserves critical geometric information typically destroyed by the limit operator, paving the way for more stable numerical simulations.

Acknowledgements.

This research was partially supported by Namirial S.p.A. and Dectarlab S.r.l.

References

- [1] Antoniotti L., Marotta C.M., Melicchio A., Papaleo M.A. (2025) The Cantor-Vitali function and infinity computing. In: Sergeyev, Y.D., Kvasov, D.E., Astorino, A. (eds.), *Numerical Computations: Theory and Algorithms 4th Int. Conf.*, Springer, LNCS Vol. 14478, pp. 232–239.
- [2] Antoniotti L., Caldarola F., Maiolo M. (2020) Infinite Numerical Computing Applied to Hilbert’s, Peano’s, and Moore’s Curves. *Mediterranean Journal of Mathematics*, vol. 17(3), 99.
- [3] Antoniotti L., Caldarola F., d’Atri G., Pellegrini M. (2020) New Approaches to Basic Calculus: An Experimentation via Numerical Computation. In: Sergeyev Y.D., Kvasov D.E. (eds.), *Numerical Computations: Theory and Algorithms 3rd Int. Conf.*, Springer, LNCS Vol. 11973, pp. 329–342.

Integro Spline Quasi-Interpolants with WENO Techniques for Discontinuous Functions

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Keywords. Integro-spline quasi-interpolation; WENO; piecewise smooth functions approximation; cell averages.

We develop a new family of nonlinear quasi-interpolation operators based on B-splines and cell average data (integro-splines) for approximating piecewise smooth functions. The proposed operators utilize Weighted Essentially Non-Oscillatory (WENO) techniques to adaptively handle discontinuities while maintaining high-order accuracy in smooth regions. For degrees $d = 2, 3, 4, 5$, we construct both linear integro-spline quasi-interpolants L^d and their nonlinear WENO counterparts W^d . We provide theoretical error estimates demonstrating that the nonlinear operators achieve optimal approximation order $\mathcal{O}(h^{d+1})$ away from discontinuities and significantly reduce oscillations near jumps, limiting the affected region to the immediate neighborhood of the discontinuity. The linear operators, in contrast, exhibit Gibbs phenomenon across a wider interval proportional to the support of the basis functions. Numerical experiments confirm the theoretical predictions and illustrate the superior performance of WENO-based integro-spline quasi-interpolation for reconstructing functions with isolated discontinuities. The proposed methods are particularly suitable for applications in image processing, conservation laws, and data compression where preservation of sharp features is essential.

References

- [1] Aràndiga F., Belda A. M., Mulet P. (2010) Point-value WENO multiresolution applications to stable image compression. *Journal of Scientific Computing*, vol. 43(2), pp. 158–182.
- [2] Aràndiga F., Donat R., López-Ureña S. (2023) Nonlinear improvements of quasi-interpolant splines to approximate piecewise smooth functions. *Appl. Math. Comp.*, vol. 448, article 127946.
- [3] Aràndiga F., Lamberti P., Remogna S. (2026) Nonlinear quartic quasi-interpolating splines for piecewise smooth function approximation. *Journal of Computational and Applied Mathematics*, vol. 474, article 116890.
- [4] Aràndiga F., Remogna S. (2026) Approximation of piecewise smooth functions by nonuniform nonlinear quadratic and cubic spline quasi-interpolants. *Mathematics and Computers in Simulation*, vol. 241, pp. 758–770.

Probability Measures in Motion: Optimal Transport, Diffusion Models, and Bayesian Optimization as a Unified Framework

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Keywords. Optimal transport; diffusion models; Bayesian optimization.

We are now witnessing a scientific revolution in which computation has become the decisive transformative resource of our era. This talk surveys interconnected paradigms at the frontier of optimization, combinatorics, probability theory, ML and applied mathematics, all understood through the evolution of probability measures. We trace the arc from Monge [1] through Kantorovich [2] to the Gromov-Wasserstein (GW) framework for comparing *metric measure spaces*, with application to structured data, graphs, and cross-space problems. [3] argued that science advances through paradigm shifts, and the rise of AI constitutes such a shift. Probabilistic numerics [4] recasts numerical methods as Bayesian inference, dissolving the boundary between computation and learning. Optimal Transport (OT) couples probability measures by minimizing transport cost, yielding the Wasserstein distance linked to gradient flows and entropic OT. Bayesian Optimization (BO) optimizes black-box functions via a probabilistic surrogate updated by an acquisition function, arising in hyperparameter tuning, architecture search, and combinatorial OT. GW removes OT’s shared-space restriction over couplings between *metric measure spaces*, enabling comparison of graphs, point-clouds, and heterogeneous data, with applications in graph classification, single-cell genomics, and cross-modal learning. GW is equivalent to QAP and NP-hard, and BO serves as a sample-efficient strategy connecting all three paradigms. OT contributes variational structure and geometric rigor; diffusion models contribute scalable stochastic dynamics; BO contributes uncertainty quantification and sequential decision-making, together forming a coherent mathematical language for the current AI revolution.

References

- [1] Monge G. (1781) Memoire sur la theorie des deblais et des remblais. *Mem. Math. Phys. Acad. Royale Sci.*, pp. 666–704.
- [2] Kantorovich L. V. (1942) On the translocation of masses. *Dokl. Akad. Nauk. USSR*, vol. 37, pp. 199–201.
- [3] Kuhn T. S. (1997) *The Structure of Scientific Revolutions* (Vol. 962). Chicago: University of Chicago Press.
- [4] Hennig P., Osborne M. A., Kersting H. P. (2022) *Probabilistic Numerics: Computation as Machine Learning*. Cambridge University Press.

Computer Analogy Method and its Application for Solving Nonlinear Kinetic Systems

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Keywords. Computer analogy method; nonlinear differential equations; Cauchy problem; kinetic systems; Boltzmann equation.

The previously proposed computer analogy method (CAM) [1] is further developed. The essence of CAM lies in the formalization of the basic properties of a digital computer, such as a fixed number of digits and the transfer from one rank to another.

To solve the Cauchy problem, a corresponding difference scheme is considered. The solution is represented as a segment of a convergent power series in powers of the step size of the independent variable (referred to as “time”).

We demonstrate the application of the method for solving nonlinear discrete kinetic systems approximating the Boltzmann equation [2]. The right-hand side of the systems is given by a quadratic form which, when combined with a value shifting function, allows reducing the solution for each variable to the sum of four terms.

The coefficients corresponding to the second and third powers of the time step can be obtained using uniformly distributed random variables, leading to a reduction in computational cost. As the time step decreases, the main contribution comes from the linear term. The limiting frequency of shifts to this linear term, as the time step approaches zero, can be interpreted as a new function through which the solution is expressed.

An analytical solution is obtained for a system of four equations, while a semi-analytical approximation is derived for a system of six equations using uniformly distributed random variables.

References

- [1] Aristov V., Stroganov A. (2012) A method of formalizing computer operations for solving nonlinear differential equations. *Applied Mathematics and Computation*, vol. 218, pp. 8083–8098.
- [2] Godunov S., Sultangazin U. (1971) On discrete models of the kinetic Boltzmann equation. *Russian Mathematical Surveys*, vol. 26(3), pp. 1–56

A Numerical Method to Solve Maxwell's Equations in Axisymmetric Singular Domains

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Keywords. Maxwell equations; singularities; axisymmetric geometry.

This work introduces a variational approach for solving the three-dimensional Maxwell equations in axisymmetric singular domains. We consider geometries generated by the rotation of a singular polygon (featuring reentrant corners or edges) around a symmetry axis. While the problem is inherently 3D, the axisymmetric assumption allows for a reduction to a 2D computational domain, while preserving the three-dimensional nature of the electromagnetic fields.

By employing Fourier analysis in the angular direction, the 3D problem is transformed into a sequence of uncoupled 2D problems indexed by the Fourier mode k . A key feature of our approach is the decomposition of the solution for each mode into a regular part and a singular part. The regular component is captured using standard finite element methods, while the singular part, which belongs to a finite-dimensional subspace, is specifically treated to account for the presence of reentrant corners and edges.

We propose a numerical strategy where the singular basis is computed for specific modes ($k = 0, \pm 1$) and a stabilization mode ($k = 2$), which effectively represents the singular behavior for all higher-order modes. The total time-dependent solution is then reconstructed through a non-stationary variational formulation. Numerical results demonstrate the capability of the method to capture the field singularities. This work generalizes the Singular Complement Method (SCM) to time-dependent 3D problems in axisymmetric configurations, see e.g. [1] and [2].

References

- [1] Assous F., Ciarlet P., Jr., Labrunie S. (2018) *Mathematical Foundations of Computational Electromagnetism*. Applied Mathematical Sciences series, vol. 198. Springer, Cham.
- [2] Assous F., Raichik I. (2020) Numerical solution to the 3D static Maxwell equations in axisymmetric singular domains with arbitrary data. *J. Comput. Meths. Appl. Maths.*, vol. 20(2), pp. 419–435.

Scattering Networks on Finite Groups

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Keywords. Wavelets on finite groups; convolutional neural networks; scattering transform; group-covariant convolutional neural networks.

We will discuss a construction of convolutional neural networks on finite groups based on the scattering networks scheme, built on top of a notion of wavelets associated to the decomposition of the regular representation of the group in terms of irreducible representations. We will prove stability, isometry, covariance and invariance for these networks. The admissibility of the weights of the network is provided in terms of explicit conditions with minor constraints.

This is a work in collaboration with M. T. Arias and E. Hernández. These results are contained in the preprint [1].

Acknowledgements.

This research was supported by the Grants PID2019-105599GB-I00, PID2022-142202NB-I00 / AEI / 10.13039/501100011033

References

- [1] Arias M.T., Barbieri D., Hernández E. (2025) Scattering Networks on Noncommutative Finite Groups. <https://arxiv.org/abs/2505.20950>.

Numerical Precision and Performance Trade-offs in Graph Neural Network Inference

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Keywords. Reduced-precision computing; graph neural networks; deep learning inference.

Reduced numerical precision is increasingly used to accelerate deep learning inference on modern computing platforms. While formats such as FP16 (Half-Precision Floating-Point) and INT8 (8-bit Integer) can significantly improve computational efficiency [1], their impact on predictive performance depends on the model architecture and the target hardware. In this work, we present a preliminary study on the effect of numerical precision in graph neural network (GNN) [2] inference. We evaluate a dense GCN-style (Graph Convolutional Network) model for graph classification using three numerical formats: FP32 (32-bit Floating Point), FP16, and INT8. The models are exported through ONNX (Open Neural Network Exchange) [3] and optimized using the TensorRT [4] inference framework. Tests are conducted on two GPU (Graphics Processing Unit) platforms: a desktop and an edge device. We report initial observations of inference latency and classification accuracy across benchmark graph datasets, highlighting the trade-offs among numerical precision, computational efficiency, and predictive performance.

References

- [1] Sze V., Chen Y.H., Yang T.J., Emer J.S. (2017) Efficient Processing of Deep Neural Networks: A Tutorial and Survey. *Proceedings of the IEEE*, vol.105(12), pp.2295–2329. DOI: 10.1109/JPROC.2017.2761740
- [2] Wu Z., Pan S., Chen F., Long G., Zhang C., Yu P.S. (2021) A Comprehensive Survey on Graph Neural Networks. *IEEE Transactions on Neural Networks and Learning Systems*, vol. 32(1), pp. 4–24. DOI: 10.1109/TNNLS.2020.2978386
- [3] ONNX (Open Neural Network Exchange). <https://onnx.ai/>
- [4] NVIDIA TensorRT. <https://developer.nvidia.com/tensorrt>

Dynamic Proximal Point Method for Constrained Minimization

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Keywords. Constrained minimization; proximal point.

Constrained optimization is a fundamental problem in numerical analysis. Constrained minimizers may be ill-conditioned or, more generally, difficult to compute with simple algorithms. To improve robustness, one may use the *proximal point* technique. However, in easy cases where such regularization is unnecessary, this approach may slow down convergence.

In the classical setting, the *proximal point* method leads to a two-level scheme: at the first level, a better-conditioned problem is solved to obtain the search direction; at the second level, this direction is used in a *backtracking* procedure to determine the step length. This strategy is effective when the first-level problem can be solved analytically, but for more complex problems it must be treated iteratively. Moreover, it requires the tuning of several problem-dependent parameters.

In this work, we replace the classical first-level scheme with a dynamic approach, in which the *proximal point* term evolves according to a pseudodynamic rule. As a result, the search direction is approximated by a single iteration of the first-level problem. In addition, the pseudodynamic evolution is used to adapt the parameters during the iterative process, with the aim of preserving robustness without significantly reducing convergence speed.

Acknowledgements. This study was carried out within the project “*SPACE IT UP! ASI contract n. 2024-5-E.0 CUP Master n. I53D24000060005*” funded by the Italian Space Agency (ASI), and the Ministry of University and Research.

References

- [1] Rockafellar R. T. (1976) Monotone operators and the proximal point algorithm. *SIAM Journal on Control and Optimization*, vol. 14, pp. 877–898.
- [2] Parikh N., Boyd S. (2014) Proximal algorithms. *Foundations and Trends in Optimization*, vol. 1(3), pp. 127–239.
- [3] De Marchi A. (2021) Augmented Lagrangian methods as dynamical systems for constrained optimization. *60th IEEE Conference on Decision and Control (CDC)*, pp. 6533–6538.

Quantifying Nonconvex Granular Entanglement Using High-Performance Computing

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Keywords. Granular media; entanglement; high-performance computing; DEM; peridynamics; nonconvexity.

Granular entanglement describes the jamming behavior that emerges from the overlap between convex hulls of nonconvex grains [3]. In this work, we identify grain-scale metrics responsible for granular entanglement in aggregates. Given an arbitrary, possibly nonconvex grain shape, we propose numerical algorithms to compute grain-scale entanglement metrics such as accessibility and angular coverage in two dimensions. We adapt the massively parallel simulation platform PeriGrain [1, 2] to bulk settling simulations, which utilizes a hybrid peridynamics-based Discrete Element Method (PeriDEM) framework [4]. We perform bulk simulations of homogeneous granular aggregates of various parametrized families of nonconvex grains and quantify the relationship between the grain-scale metrics and the bulk entanglement indicators such as settling height. Our work allows for precise grain shape engineering to achieve a desired homogenized bulk property associated with structural resilience of granular aggregates.

Acknowledgements.

This research was supported by the Mentored Advanced Project (MAP) program at Grinnell College. High-performance computing resources at Grinnell College were utilized.

References

- [1] Bhattacharya D., Lipton R. (2023) Simulating grain shape effects and damage in granular media using PeriDEM. *SIAM Journal on Computing*, vol. 45(1), B1–B26.
- [2] Bhattacharya D., Lipton R. (2025) Macroscopic effects of intraparticle fracture, grain topology and shape on vehicle dynamics and mobility over gravel road beds. *Granular Matter*, vol. 27(2).
- [3] Gravish N., Franklin S.V., Hu D.L., Goldman D.I. (2012) Entangled Granular Media. *Physical Review Letters*, vol. 108(20), 208001.
- [4] Jha P.K., Desai P.S., Bhattacharya D., Lipton R. (2021) Peridynamics-based discrete element method (PeriDEM) model of granular systems involving breakage of arbitrarily shaped particles. *Journal of the Mechanics and Physics of Solids*, vol. 151, 104376.

Anonymization Methods for Photographic Imaging Diagnostics in Scoliosis Patients

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Keywords. Anonymization; scoliosis; photo.

Scoliosis, typically defined as a lateral curvature of the spine in the frontal plane, is diagnosed by standing posterior-anterior (PA) X-rays of the full spine, which are used to measure the Cobb angle. Normally, a Cobb angle of 10 is considered a minimum angulation to define scoliosis. In addition to radiographic measurements, the diagnosis of scoliosis relies on physical examinations such as the Adams test, during which the angle of trunk rotation (ATR) is measured. However, these traditional tools have limitations: Firstly, the cumulative radiation dose exposes younger patients to biological risks. Secondly, the measurements are operator-dependent and subject to interoperator/intraoperator variation. For this reason, new tools that claim to automatically diagnose scoliosis by photographic analysis are becoming popular. But photography captures sensitive biometric data, which raises significant privacy concerns. In this paper, we present the main anonymization methods applied to orthopedic clinical photography, evaluating the trade-off between protecting the patient's identity and preserving scoliosis markers. In particular, we will compare classic image anonymization techniques, such as blurring and pixelation, with more advanced solutions based on deep learning, such as MediaPipe. We will also compare these with image anonymization methods that use generative techniques, such as GANs [1, 2]. Furthermore, deep learning (DL) algorithms require a vast amount of data for training. To resolve the data bottleneck problem we examine techniques such as data augmentation to generate synthetic cases, thereby augmenting the dataset and improving the training phase of deep learning systems [3].

References

- [1] Ban B., Lee H. (2024) FAKER: Full-body anonymization with human keypoint extraction for real-time video deidentification. *arXiv preprint arXiv:2408.11829*.
- [2] Lee J. H., You S. J. (2024) Balancing privacy and accuracy: Exploring the impact of data anonymization on deep learning models in computer vision. *IEEE Access*, vol. 12, pp. 8346–8358.
- [3] Zhou Z., Guo Y., Tang R. (2024) Privacy enhancing and generalizable deep learning with synthetic data for mediastinal neoplasm diagnosis. *NPJ Digit. Med.*, vol 7, article 293.

A Multimodal Computational Framework for Robust Similarity-based Drug Repurposing

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Keywords. Multimodal molecular representations; dimensionality reduction; similarity learning.

Artificial Intelligence (AI) and computational mathematics are transforming scientific research. Within this context, the representation and processing of molecular data have become a key challenge for AI-based drug discovery and repurposing. This study presents a mathematical and AI-driven pipeline for learning molecular representation and analyzing compound similarity. We explore and integrate multiple complementary representations, including fingerprint-based, image-based, and graph-based encodings, to improve the expressiveness and robustness of molecular similarity estimation. Image-based representations are extracted via transfer learning using a ResNet50 convolutional neural network enhanced by a Group Equivariant Convolutional Network to ensure geometric equivariance to rotations and reflections. Concerning graph-based representations, a Graph Neural Network is trained on millions of molecules represented as graphs to learn embeddings. These representations are then combined through an ensemble strategy to leverage different chemical information spaces. The computational workflow integrates numerical analysis, dimensionality reduction, and machine learning techniques: Non-Negative Matrix Factorization and Singular Value Decomposition are used to project high-dimensional feature spaces; Gaussian Mixture Models identify molecular clusters; and cosine similarity metrics rank compounds according to their proximity in the latent space. As a case study, the pipeline was applied to drug repurposing for inhibitors of the oncogenic RAS G12C mutation. Starting from an initial set of 271 compounds, the ensemble approach consistently identified a subset of 20 molecules as the most promising candidates for further investigation.

References

- [1] Muegge I., Mukherjee P. (2016) An overview of molecular fingerprint similarity search in virtual screening. *Expert Opin. Drug Discov.*, vol. 11(2), pp. 137–48.
- [2] Cohen T. et al. (2016) Group Equivariant Convolutional Networks.
<https://arxiv.org/abs/1602.07576>
- [3] Corso G. et al. (2024) Graph neural networks. *Nat. Rev. Methods Primers*, vol. 4(1), 17.

Hierarchical Multilevel Spline Quasi-Interpolation

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Keywords. Quasi-interpolation; hierarchical splines; discontinuities; non-linear operators.

In [2] the authors studied how to extend quasi-interpolants (QIs) defined on tensor-product spline spaces to hierarchical ones [1]. The approach chosen there, which requires the QIs to be projectors, allows to combine the properties of classical QIs with the possibility of local refinement offered by hierarchical splines. Here we present an extension of the hierarchical QI construction to the case where the initial QIs are not projectors (and are defined on a set of generators). This is essentially done by employing a multilevel approach: at each level, the QI is updated by adding the local approximation of the residual. In particular, we apply our construction to the WENO QIs for piecewise-smooth functions introduced in [3].

Acknowledgements.

This research was supported by Gruppo Nazionale per il Calcolo Scientifico - Istituto Nazionale di Alta Matematica (Italy) through the GNCS 2025 project “Tecnologie spline efficienti per l’analisi isogeometrica e l’approssimazione di dati”.

References

- [1] Giannelli C., Jüttler B., Speleers H. (2014) Strongly stable bases for adaptively refined multilevel spline spaces. *Adv. Comput. Math.*, vol. 40, pp. 459–490.
- [2] Speleers H., Manni C. (2016) Effortless quasi-interpolation in hierarchical spaces. *Numer. Math.*, vol. 132, pp. 155–184.
- [3] Aràndiga F., Remogna S. (2023) Nonlinear 2D C^1 quadratic spline quasi-interpolants on triangulations for the approximation of piecewise smooth functions. *Axioms*, vol. 12, 1002.

Recent Advances in Fractional Hamiltonian Boundary Value Methods

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Keywords. Fractional differential equations; Caputo derivative; FHBVMs.

Over the past two decades, the mathematical modelling of Fractional Differential Equations (FDEs) has attracted growing attention and expanded significantly across various scientific fields. In this context, finding analytical solutions is often more challenging than for classical ordinary differential equations, while accurate and reliable numerical methods can be hindered by the potential nonsmoothness of the solution and/or vector field at the initial time. Moreover, the nonlocality of the differential operator and the persistence of the intrinsic memory term can render long-time simulations computationally demanding. To mitigate these issues, the class of Runge-Kutta type methods, known as Fractional Hamiltonian Boundary Value Methods (FHBVMs), is presented, covering its design, development and analysis. In particular, a novel extension [1] is discussed, allowing for a mixed graded/uniform mesh for time step selection and resulting in an updated version of pre-existing Matlab codes. Numerical experiments show that this approach is especially effective for problems having nonsmooth vector field/solution at the initial time, with solution of oscillatory type, achieving higher accuracy in reproducing the initial nonsmooth behavior, while maintaining efficiency over long time periods. Finally, a generalisation to fractional multi-order problems [2] is introduced and applied to model predator-prey dynamics with intraguild predation, effectively accounting for potentially different rates of change of the populations with respect to their own time history.

Acknowledgements.

This research was partially supported by INdAM-GNCS.

References

- [1] Brugnano L., Gurioli G., Iavernaro F., Vikerpuur M. (2025) Analysis and implementation of collocation methods for fractional differential equations. *Journal of Scientific Computing*, Vol. 104, id. 92.
- [2] Brugnano L., Gurioli G., Iavernaro F., Vikerpuur M. (2026) Recent advances in the numerical solution of multi-order fractional differential equations. arXiv preprint arXiv:2512.04737.

Explainable and Uncertainty-Aware Modelling in Computational Pathology via Evidential Multi-Layer Perceptron

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Keywords. Explainable AI; evidential learning; digital pathology.

The analysis of high-dimensional structured data in AI-based models requires rigorous approaches to interpretability and uncertainty. In computational pathology, whole-slide images (WSIs) are high-resolution scans divided into spatial tiles, with supervision typically only at the WSI level, introducing epistemic uncertainty and motivating mathematically grounded aggregation for robust WSI-level prediction.

In this study, we propose a computational framework for uncertainty-aware classification of WSIs based on evidential learning and spatial activation maps. Each WSI is represented as a set of embedded tiles extracted using UNI2, a self-supervised vision encoder for computational pathology [1]. Selected features train an Evidential Multi-Layer Perceptron (EvMLP) for tile-level prediction, outputting Dirichlet parameters from which predictive probabilities and epistemic uncertainty are derived. The model is optimized with an evidential loss combining expected cross-entropy and Kullback-Leibler regularization. WSI-level predictions use a Reliability-Weighted Aggregation (RWA) that downweights uncertain tiles, while spatial back-projection of tile-level evidence generates interpretable maps of discriminative regions and uncertainty.

As a case study, the framework predicted immunotherapy response from 59 pre-treatment H&E WSIs of MSI-CRC biopsies (337,293 tiles) and localized responder-like regions. Cross-validation achieved a WSI-level AUC of 0.87, with 0.75 on the independent test set, and spatial maps strongly concorded with blinded expert annotations (Concordance Index = 0.73) in non-responder regions.

Acknowledgements.

This research was supported by the European Union - NextGenerationEU under Project Code PNRR-MCNT-2023-12377189.

References

- [1] Chen R. J., Ding T., Lu M. Y., et al. (2024) Towards a general-purpose foundation model for computational pathology. *Nature Medicine*, vol. 30(3): pp. 850–862.

Geometric Foundations and Local Dynamics in Water Distribution Networks

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Keywords. Graph theory; water distribution networks; resilience.

This talk explores the integration of Graph Theory and advanced mathematical frameworks to model and optimize Water Distribution Networks (WDNs). While global performance metrics are standard in hydraulic engineering, this presentation introduces a specialized framework designed for fine-grained local analysis. By leveraging discrete geometry, linear algebra and vector analysis, we define new local indices that characterize the importance and vulnerability of individual nodes and pipes within the network architecture.

A central theme of the discussion is the characterization of resilience. We examine how local structural properties influence the system's ability to withstand failures, alongside an investigation into invariance problems—identifying properties that remain consistent despite topological perturbations or fluctuating demand patterns. Finally, the talk bridges the gap between theoretical combinatorics and environmental safety. We adapt classical mathematical studies regarding random walks and paths in grids under variable conditions to the physical reality of WDNs. This allows us to model the propagation of pollutants with high precision, linking path-finding algorithms to the stochastic nature of contaminant transport.

References

- [1] Caldarola F., Carini M., Maiolo M. (2025) A Simple Mathematical Solution to an Invariance Problem in Water Networks. In: Sergeyev, Y.D., Kvasov, D.E., Astorino, A. (eds.), *Numerical Computations: Theory and Algorithms 4th Int. Conf.*, Springer, LNCS Vol. 14477, pp. 192–199.
- [2] Caldarola F., Maiolo M. (2020) Algebraic tools and new local indices for water networks: some numerical examples. In: Sergeyev, Y.D., Kvasov, D.E. (eds.), *Numerical Computations: Theory and Algorithms 3rd Int. Conf.*, Springer, LNCS, vol. 11973, pp. 517–524.
- [3] Caldarola F., Maiolo M. (2021) A mathematical investigation on the invariance problem of some hydraulic indices. *Applied Mathematics and Computation*, vol. 409, 125726.

Iterative Geometric Models and Fractal Dynamics for Coastal Morphologies

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Keywords. Fractal geometry; iterative systems; coastal profiles.

This talk presents an advanced mathematical approach to modeling coastal lineaments through iterative processes based on plane curves. Drawing from the framework established in recent research, we move beyond Euclidean approximations to embrace the intrinsic fractal nature of maritime boundaries. By employing recursive functional systems and iterative generators, we construct geometric models capable of capturing the multi-scale complexity of coastal erosion and accretion. These models provide a rigorous basis for calculating the fractal dimension of specific shorelines, serving as a critical indicator of geomorphological complexity.

A significant portion of the presentation is dedicated to the dynamics of coastal bays. We analyze how iterative mathematical structures can simulate the equilibrium of bay shapes, often governed by logarithmic spirals or higher-order geometric constraints. By integrating dynamical systems with boundary condition variables, we demonstrate how these curves adapt to long-term sediment transport and wave diffraction patterns. This intersection of pure geometry and environmental fluid dynamics offers a robust predictive tool for coastal management. Ultimately, the talk highlights how iterative mathematical refinement transforms static coastal mapping into a dynamic, scale-invariant representation of natural interfaces.

References

- [1] Caldarola F., Carini M., Costarella A., de Raffe G., Maiolo M. (2025) Physical-Mathematical Modeling and Simulations for a Feasible Oscillating Water Column Plant. *Mathematics*, vol. 13, 2219.
- [2] Caldarola F., Carini M., Maiolo M., Papaleo M. A. (2024) Iterative Mathematical Models Based on Curves and Applications to Coastal Profiles. *Mediterranean Journal of Mathematics*, vol. 21, 178.
- [3] Huynh H.N., Chew L.Y. (2011) Arc-fractal and the dynamics of coastal morphology. *Fractals*, vol. 19, pp. 141–162.

What Can We Learn from the 2009 Giampilieri Disaster?

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Keywords. Giampilieri; debris/mud flows; cellular automata; hazard assessment; numerical simulation; safety works; retro-innovation.

Concretization (very costly) has been one of the most common measures used to secure areas ruined by debris/mud/granular flows. In a context of climate change, such a solution becomes unfeasible given the growing number of areas at risk. Here, we critically analyze, not only from a numerical simulation perspective but also from a historical and sociological perspective, a paradigmatic case of future events: the 2009 Giampilieri Superiore catastrophe [1], which resulted in 37 deaths, many injuries, and severe infrastructure damage. Around 200 million euros were used to secure the territory, building a huge “containment wall/drainage channel” to intercept and convey possible debris/mud flows and other minor containment walls.

The macroscopic Cellular Automata SCIDDICA (urban version) model was applied to simulate the disaster. The simulations of this digital twin were excellent and permitted verification of the efficiency of the safety works [1]. The historical analysis of the causes of soil degradation identified a retro-innovation [2] strategy for soil recovery; a series of new simulations confirmed that the same initial triggering conditions of the 2009 disaster generate harmless debris flows. The sociological analysis allowed us to define the conditions for carrying forward the retro-innovation process, allowing recovery of previous land use at costs lower than those of concreting.

References

- [1] Di Gregorio S., Lupiano V., Forno F., Calidonna C. R., Catelan P., Chidichimo F. (2025) A trans-disciplinary analysis of the 2009 catastrophe in Giampilieri Superiore by land use evolution over time. *Earth Syst. Environ.*, vol. 9, pp. 505–517.
- [2] Stuiver M. (2006) Highlighting the retro side of innovation and its potential for regime change in agriculture. In: Marsden T., Murdoch J. (eds.) *Between the Local and the Global*. Research in Rural Sociology and Development, vol. 12. Emerald Group Publishing Limited, Leeds, pp. 147–173.

Neural Network Design for the Stable Selection of the HP-Spline Frequency Parameter

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Keywords. HP-splines; adaptive parameter estimation; Artificial Neural Networks stability.

We explore the use of artificial neural networks for the stable and controlled data-driven selection of the frequency parameter in hyperbolic polynomial penalized splines (HP-splines). This parameter α , which defines the underlying spline space, is crucial to adapt the model to exponential patterns in the data, such as those encountered in signal processing [1]. We investigate the theoretical foundations of deep neural network architectures [2], with the aim of establishing a connection between classical spline-based regression and modern data-driven learning approaches. We introduce a two-stage learning architecture. Firstly we establish several upper bounds on the predictive accuracy by ReLU networks, based on the complexity of the network and the smoothness order of specific function classes where α function is well described. Then, we derive a wavelet-based Lipschitz constant of the full system. The resulting stability theorem is stated formally. Numerical experiments confirm the reliability of the predictive framework, compared with a deterministic algorithm [3], and the enhanced explainability of neural architectures when designed by combining approximation accuracy, stability analysis, and complexity control.

References

- [1] Bruni V., Campagna R., Vitulano D. (2025) Multicomponent signals interference detection exploiting HP-Splines frequency parameter, *Applied Numerical Mathematics*, vol. 209.
- [2] Yarotsky D. (2017) Error bounds for approximations with deep ReLU networks. *Neural networks*, vol. 94, pp. 103–114.
- [3] Campagna R., Conti C., Cuomo S. (2023) A linear algebra approach to HP-splines frequency parameter selection. *Applied Mathematics*, vol. 458.

Gromov-Monge as a Sequence of Linear Programming Problems

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Keywords. Optimal transport theory; Gromov-Monge; Gromov-Wasserstein.

The Gromov-Monge (GM) problem is increasingly becoming relevant, especially within the Machine Learning community, because it permits quantifying the structural distance between two data sets which lay into two different metric spaces, as well as the same data set represented through two completely different sets of features. Looking at the two data sets as two empirical discrete distributions leads to the concept of *measure metric space* (**mm-spaces**): while a metric space is a pair (X, d_X) – with $X \subset \mathbb{R}^{h_X}$ a non-empty, separable, and completely metrizable space (i.e. a *Polish* space) and $d_X : X \times X \rightarrow \mathbb{R}$ a *distance metric* generating the geometry of the space – an mm-space is a triple (X, d_x, μ_x) where the further component μ_x is a probability measure. Roughly, an mm-space is a metric space equipped with a probability distribution. This extension allows to exploit the Optimal Transport (OT) theory [1] to solve computability issues of the Gromov-Housdorff distance between two metric spaces, leading to the so-called Gromov-Wasserstein (GW) distance and, in the case of two mm-spaces defined by two data sets, to its GM formulation [2,3]. Unfortunately, GM is a Quadratic Assignment Problem (QAP), an NP-hard problem that is still computationally expensive to solve even for small instances [4]. We prove that there exists a *parametrized* transformation of the two mm-spaces into two probability measures over the same metric space, leading to a linear programming problem whose solution is always feasible, but not optimal, for GM. Thus, solving GM translates into searching for the parameter’s value minimizing GM. Contrary to state-of-the-art approaches, our method does not rely on any approximation (e.g., linearization, entropic regularization, slicing, etc.) of the original problem.

References

- [1] Peyré G., Cuturi M. (2019) *Computational optimal transport: with applications to data science*. Now Foundations and Trends. NOW Publishers.
- [2] Memoli F., Needham T. (2024) Comparison results for Gromov-Wasserstein and Gromov-Monge distances. *ESAIM: Control, Optimisation and Calculus of Variations*, vol. 30, article 78.
- [3] Dumont T., Lacombe T., Vialard F. X. (2025) On the existence of Monge maps for the Gromov-Wasserstein problem. *Foundations of Computational Mathematics*, vol. 25(2), pp. 463–510.
- [4] Pitsoulis L., Pardalos P. M. (2024) Quadratic assignment problem. In: *Encyclopedia of Optimization*, Cham: Springer International Publishing.

A Parallel Deep Learning Ensemble for Multi-Subject SSVEP Classification

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Keywords. Brain-computer interfaces; steady-state visually evoked potentials; parallel ensemble learning; deep learning.

Accurate classification of electroencephalographic (EEG) signals in multi-subject Brain-Computer Interfaces (BCIs) requires computational approaches able to cope with high inter-subject variability, heterogeneous patterns, and non-stationary signal distributions [1]. In this context, relying on a single monolithic model may limit the ability to capture subject-dependent and frequency-specific dynamics effectively.

To address this challenge, we propose a computational framework for SSVEP-based BCIs based on a Mixture-of-Experts (MoE) strategy [2], in which multiple deep base learners operate in parallel and a gating mechanism adaptively combines their outputs. Each expert is implemented as a Multi-Level ResNet and is designed to learn complementary representations from one or multiple subjects, enabling a specialized processing of EEG responses.

The proposed architecture is a parallel deep learning paradigm for robust decision-making in complex neurophysiological settings, where learner diversity is exploited as a resource rather than treated as noise. Our solution fits within current efforts in computational science to develop modular, scalable, and efficient intelligent systems for analyzing complex real-world phenomena.

References

- [1] Rizzo L., Cicirelli F., D’Amore F., Gentile A. F., Guerrieri A., Vinci A. (2025) Using Brain-Computer Interface in Cognitive Buildings: a Real-Time Case Study. In: *5th IEEE International Conference on Human-Machine Systems (ICHMS 2025)*, Abu Dhabi, United Arab Emirates, pp. 433–436, IEEE. DOI: 10.1109/ICHMS65439.2025.11154229.
- [2] Caviglione L., Guarascio M., Pisani F. S., Zuppelli M. (2024) A Few to Unveil Them All: Leveraging Mixture of Experts on Minimal Data for Detecting Covert Channels in Containerized Cloud Infrastructures. In: *IEEE European Symposium on Security and Privacy Workshops (EuroS&PW 2024)*, Vienna, Austria, pp. 731–739, IEEE. DOI: 10.1109/EUROSPW61312.2024.00090.

Fourier Transform of Vector-Valued Graph Signals

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Keywords. Graph signal processing; graph Fourier transform; vector-valued graph signals; Banach spaces; Hilbert spaces.

In the last decades, graph theory has undergone significant development from both theoretical and practical points of view, spanning several branches of Mathematics, such as Algebra, Geometry, Analysis and Numerical Analysis. It has become a fundamental branch of Mathematics that models relationships between objects.

Classical Graph Signal Processing provides a robust framework for analyzing signals on irregular domains, utilizing the graph Fourier transform as cornerstone for spectral analysis and filtering. As data structures grow in complexity, there is an increasing need to handle multi-dimensional information. We propose a generalization of the Graph Signal Processing framework by introducing vector-valued graph signals which take values in arbitrary Banach spaces.

In this work, we define the fundamental operators of time-frequency analysis within this broader setting, including the graph Fourier transform, convolution, and translation operators for multidimensional structured data, enabling spectral analysis of multiple signals on graphs simultaneously.

A key contribution of this work is the derivation of operator norm estimates of the graph Fourier transform, and the establishment of graph-theoretic versions of classical uncertainty principles. We demonstrate how these results depend on the choice of the orthonormal basis and on the underlying L^p norms.

This framework facilitates the study of inter-signal correlations, providing a flexible and mathematically grounded environment for analyzing multivariate time-series and time-varying signals on complex networks.

References

- [1] Ghandehari M., Guillet D., Hollingsworth K. (2021) Gabor-type frames for signal processing on graphs. *Journal of Fourier Analysis and Applications*, vol. 27, article 25.
- [2] Hammond D.K., Vandergheynst P., Gribonval R. (2011) Wavelets on graphs via spectral graph theory. *Applied and Computational Harmonic Analysis*, vol. 30(2), pp. 129–150.
- [3] Sandryhaila A., Moura J. M. F. (2013) Discrete signal processing on graphs. *IEEE Transactions on Signal Processing*, vol. 61(7), pp. 1644–1656.
- [4] Shuman D.I., Ricaud B., Vandergheynst P. (2016) Vertex-frequency analysis on graphs. *Applied and Computational Harmonic Analysis*, vol. 40(2), pp. 260–291.

Algorithms Inside HEMSim: A Thermochemical Framework for the Numerical Simulation of Detonation and Combustion Phenomena

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Keywords. Detonation modeling; chemical equilibrium; nonlinear optimization; JWL equation of state; numerical algorithms.

HEMSim, a unified thermochemical software framework for the numerical simulation of detonation and combustion processes in energetic materials, is presented. The code is designed to consistently treat ideal detonation (using Chapman-Jouguet theory), isobaric combustion (rocket-like configurations), and isochoric combustion (closed-vessel systems) within a single numerical architecture.

From a computational standpoint, the core of HEMSim relies on the formulation of chemical equilibrium as a constrained minimization problem of the appropriate thermodynamic potential (Gibbs or Helmholtz free energy). The equilibrium state is computed through an interior-point optimization strategy ensuring numerical stability even in the presence of multiphase products and partial metal reactions. A novel stopping criterion avoids the need for ad-hoc phase adjustments, improving reliability.

The overall software architecture is modular and object-oriented, enabling extensibility toward advanced reactive flow solvers and coupling with hydrocodes. Validation against established thermochemical tools and experimental benchmarks demonstrates accuracy and computational efficiency.

References

- [1] Caridi Y., Cucuzzella A., Berrone S. (2024) HEMSim: a new MATLAB software to simulate the behavior of highly energetic materials upon Chapman–Jouguet hypothesis. *Journal of Energetic Materials*. DOI: 10.1080/07370652.2024.2446906
- [2] Caridi Y., Cucuzzella A., Berrone S., Vicini F. (2025) A Box-Bounded Non-Linear Least Square Minimization Algorithm with Application to the JWL Parameter Determination in the Isentropic Expansion for Highly Energetic Material Simulation. *Algorithms*. DOI: 10.3390/a18060360
- [3] Caridi Y., Cucuzzella A., Berrone S. (2025) Inside HEMSim algorithms: an interior-point-based method for non-linear constrained optimization applied to chemical equilibrium in highly energetic material simulations. *Journal of Energetic Materials*. DOI: 10.1080/07370652.2025.2597021

A Computational Approach for Musical Score Analysis

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Keywords. Graph theory; numerical methods; musical score analysis.

Quantitative analysis is not conventionally employed in the musical area. Recent studies [1–4] demonstrate that scientific calculus offers significant advantages for music analysis, style recognition, and generative prediction too. In the present study the application of Graph Theory for decoding digital musical scores is adopted to automate extraction of melodic-textual or harmonic-textual data, facilitating a unified processing of both sequential and structural information. All musical elements are suitably represented through graph structure, making it possible to conduct a quantitative analysis of the compositional characteristics of a single score or a comprehensive set of scores. The use of Graph Neural Networks, or hybrid architectures incorporating convolutional layers or autoencoders, simplify the extraction of latent structures for unsupervised stylistic clustering and supervised pattern classification. Furthermore, the proposed model enables musical forecasting through predictive inference on these representations, improving interpretability of stylistic structures and increasing robustness in predictive tasks, while significantly reducing training times.

Acknowledgements. The authors are members of the INdAM research group GNCS.

References

- [1] Licciardi S., Macchione D., Francomano E., Caronna E. (2025) From the Pentagon to the Graph Encoded Path through Rossini’s Notes. In: Proceedings of the *ICNAAM 2025 - International Conference of Numerical Analysis and Applied Mathematics*, Heraklion (Greece).
- [2] Licciardi S., Caronna E., Francomano E. (2026) A Bridge Between Science and Compositive Structure: A Computational Approach Based on Neural Networks. In: Proceedings of the *ATMA 2026 - Approximation: Theory, Methods and Application*, Rende (Italy).
- [3] Moss F.C., Neuwirth M., Harasim D., Rohrmeier M. (2019) Statistical characteristics of tonal harmony: A corpus study of Beethoven’s string quartets. *PLoS ONE*, vol. 14, article e0217242.
- [4] Kulkarni S., David S.U., Lynn C.W., Bassett D.S. (2024) Information content of note transitions in the music of J. S. Bach. *Physical Review Research*, vol. 6, article 013136.

A Presheaf Framework for Grossone Arithmetic

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Keywords. Grossone; presheaf theory; category theory; numeral systems; foundations of mathematics.

Grossone-based formalisms aim to provide an explicit treatment of infinite and infinitesimal quantities within an actual system of numeration. A natural question is whether such formalism can also be described in structural terms, in a way that preserves its computational and conceptual features. Category theory, and presheaf theory in particular, offers a rich, albeit flexible language for this purpose, as it allows the organization of the arithmetic data in a local fashion, together with restriction maps encoding changes of context. Globally meaningful quantities can be interpreted as coherent compatible families. In this paper we introduce a presheaf-based approach to Grossone arithmetic, where the central idea is to model the Grossone as a global element of a presheaf of local algebraic structures, so that its arithmetic meaning is determined by its coherence across contexts.

The paper begins by introducing a category of finite arithmetic contexts, represented by finite subsets of the positive integers, and uses it as the base for a presheaf of local algebraic structures. We assign to each context a polynomial semiring, so that the coefficients available at each stage reflect the finite divisibility information encoded by that context. Within this presheaf, Grossone can be modeled as a distinguished global element. The first part of the paper is devoted to setting up this construction and showing that the resulting family is compatible with the restriction maps, whereas the second part discusses how arithmetic properties of the Grossone are expressed through this presheaf structure.

References

- [1] Sergeyev Y.D. (2017) Numerical infinities and infinitesimals: Methodology, applications, and repercussions on two Hilbert problems. *EMS Surveys in Mathematical Sciences*, vol. 4(2), pp. 219–320.
- [2] Tohmé F., Caterina G., Gangle R. (2022) Observability in the Univalent Universe. *Mediterranean Journal of Mathematics*, vol. 19, article 216. DOI: 10.1007/s00009-022-02121-3
- [3] Gangle R., Tohmé F., Caterina G. (2025) Multicombinators as Observable Presheaves. *Soft Computing*, vol. 29, pp. 2695–3673. DOI: 10.1007/s00500-025-10636-6

A Hybrid Graph-Based Framework for Malware Evolution Analysis

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Keywords. Graph robustness; malware evolution; structural perturbations.

The rapid proliferation of malware variants, potentially accelerated by generative AI, challenges the scalability of traditional signature-based detection and boosts the development of methods able to model and anticipate the evolution of malicious software. A natural representation of this process is through graphs [2], where nodes encode malware samples or feature sets and edges describe similarity, transformation, or lineage relations within a malware family. While graph neural networks and related neural methods are effective tools for learning from such relational structures [1], their use in cybersecurity can benefit from a deeper analysis of the structural robustness [3] and fragility of the underlying networks.

We propose a hybrid framework for the analysis of malware-evolution that combines structural graph-analysis techniques with neural learning models. In particular, we investigate robustness- and fragility-oriented network indicators to identify critical nodes, edges, and structural patterns governing the evolution of malicious software, and we integrate these structural signals into a neural prediction framework for detection and mitigation tasks.

The goal is twofold: improving predictive performance in graph-based cybersecurity settings and supporting the identification of the most influential samples or graph regions whose disruption may hinder malware evolution.

Acknowledgements. This research was partially funded by Project PERUN (101225653) - Protecting Sensitive Cyber Ecosystems from Upcoming Next Generation and AI-generated Malware Threats under the Horizon Europe Program funded by the European Union.

References

- [1] Socievole A., Pizzuti C. (2023) Comparative evaluation of strategies for improving the robustness of complex networks. *Appl. Netw. Sci.*, vol. 8, article 43.
- [2] Kinable J., Kostakis O. (2011) Malware classification based on call graph clustering. *J. Comput. Virol.*, vol. 7(4), pp. 233–245.
- [3] Wang B., Jia J., Cao X., Gong N. Z. (2021) Certified robustness of graph neural networks against adversarial structural perturbation. In: *Proceedings of the 27th ACM SIGKDD Conference on Knowledge Discovery & Data Mining*, pp. 1645–1653. ACM: New York.

Enhancing Classical Methods for Initial Value Problems by Radial Basis Functions

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Keywords. Radial basis functions; initial value problems; absolute stability.

Classical numerical methods for first-order initial value problems, such as Euler’s method, the Midpoint method, Adams-Bashforth formulas, and Adams-Moulton predictor-corrector schemes, are typically constructed through polynomial interpolation. As a consequence, their order of accuracy is fixed a priori and cannot adapt to the local smoothness of the exact solution. This work investigates an alternative framework based on radial basis function (RBF) interpolation, with the aim of improving the accuracy of classical time-stepping methods while preserving their structural simplicity.

The proposed approach considers two families of kernels: the multiquadric (MQ) RBF and a C^4 -smooth Matérn (MA) RBF. In both cases, the key ingredient is the shape parameter, which introduces an additional degree of freedom absent in polynomial schemes. By expanding the local truncation error in Taylor series with respect to the step size, the error coefficients can be expressed as functions of the shape parameter. Optimal values are then selected by forcing the leading truncation-error terms to vanish, thereby increasing the order of accuracy by one or more orders whenever the resulting algebraic conditions are feasible.

The analysis is carried out for modified versions of Euler’s method, the Midpoint method, two- and three-step Adams-Bashforth methods, and one- and two-step Adams-Moulton methods. The MQ-based formulation revisits known RBF enhancements of classical schemes, while the MA-based formulation extends the same methodology to kernels with finite smoothness and controlled regularity. In both settings, polynomial interpolation is recovered as a limiting case when the shape parameter tends to zero, showing that the modified methods inherit at least the convergence behavior of their classical counterparts.

A further part of the study concerns absolute stability. Finally, numerical experiments confirm the theoretical analysis and show that the RBF-modified methods can deliver a clear improvement in accuracy over standard polynomial-based integrators.

Acknowledgements.

This research was supported by the Grant For Internationalization (GFI) 2025 Project funded by the University of Turin.

Data-Driven Approximation of High-Frequency Wave Fields: A Transfer Operator Approach

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Keywords. High-frequency wave fields; extended dynamic mode decomposition; data-driven methods.

The accurate prediction of high-frequency mechanical or electromagnetic wave fields in complex structures or environments is a challenging task with many important industrial applications. For example, modelling noise and vibration in lightweight electric vehicles or modelling the coverage of 6G and 7G wireless networks, which requires wave predictions at increasingly high frequencies. Such phenomena are characterised by highly oscillatory solutions, for which standard numerical approximations become prohibitively expensive as the frequency is increased. Energy-based approaches, including Statistical Energy Analysis and ray tracing, offer computationally viable alternatives under certain simplifying assumptions. Dynamical Energy Analysis (DEA) bridges the gap between these methods by describing the evolution of ray densities in phase-space via a transfer operator [1]. DEA provides a ray-based approach that, unlike conventional ray tracing, is not limited by the reflection order.

This work presents a data-driven implementation of DEA using Extended Dynamic Mode Decomposition (EDMD) [2]. Phase-space data are partitioned into locally smooth clusters, within which the transfer operator is efficiently approximated. Improved accuracy is achieved by employing Legendre polynomial approximations within these data clusters and/or through an additional sub-division of the phase-space. This yields a versatile and accurate DEA framework that is made tractable through the data-driven construction of the transfer operator.

Acknowledgements.

We gratefully acknowledge support from the EPSRC grant UKRI2393.

References

- [1] Tanner G. (2009) Dynamical energy analysis — determining wave energy distributions in vibro-acoustical structures in the high frequency regime. *J. Sound Vib.*, vol. 320, pp. 1023–1038.
- [2] Williams M. O., Kevrekidis I. G., Rowley C. W. (2015) A Data-Driven Approximation of the Koopman Operator: Extending Dynamic Mode Decomposition. *Journal of Nonlinear Science*, vol. 25, pp. 1307–1346.

Quantum-Enhanced Inference of Conditional Future Probabilities Beyond Quadratic Speedup

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Keywords. Quantum computation; quantum information; quantum simulation.

Accurately predicting the likelihood of high-impact events, such as earthquakes or financial “black swans”, is a vital yet challenging task. As the stochastic systems governing these events grow in complexity, their predictive models require increasing amounts of memory to track past data. Consequently, the accuracy of our estimates is fundamentally constrained by a dual bottleneck: computational limits that restrict sampling error and memory limits that induce systematic bias. Traditional quantum algorithms – such as quantum amplitude estimation [1] – address only the former and become untenable in near-term regimes where quantum memory is at a premium. Here, we offer quantum algorithmic means to reduce systematic bias while maintaining a quadratic speed-up. We demonstrated superior memory-to-bias trade-offs, estimating future probabilities with significantly less distortion than classical counterparts under identical memory constraints. These results advance the timeline for the practical deployment of quantum technologies and illustrate the unique benefits of quantum systems in simulating processes of growing complexity.

Acknowledgements.

This work is supported by the National Research Foundation of Singapore through the NRF Investigatorship Program (Award No. NRF-NRFI09-0010), the National Quantum Office, hosted in A*STAR, under its Centre for Quantum Technologies Funding Initiative (S24Q2d0009) and the Singapore Ministry of Education Tier 1 Grant RT4/23. A.G. is supported by EPSRC DTP studentship EP/W524347/1 2932638. T.J.E. is supported by the University of Manchester Dame Kathleen Ollerenshaw Fellowship. C.Y. is funded by Schmidt Sciences, LLC.

References

- [1] Brassard G., Hoyer P., Mosca M., Tapp A. (2002) Quantum amplitude amplification and estimation. *Contemporary Mathematics*, vol. 305, pp. 53–74.
- [2] Gu M., Wiesner K., Rieper E., Vedral V. (2012) Quantum mechanics can reduce the complexity of classical models. *Nat Commun*, vol. 3, article 762. <https://doi.org/10.1038/ncomms1761>

Low Synchronization Minimal Residual Methods and Their Mixed Precision Variants on GPUs

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Keywords. Minimal residual; low synchronization; mixed precision.

Low synchronization algorithms have received considerable attention due to their importance in large-scale scientific and engineering computations. In this talk we study the communication-reducing minimal residual methods for solving nonsymmetric sparse linear systems. Our focus is on the low-synchronization orthogonalization process, for which we show that the constants appearing in the theoretical bounds do not significantly compromise numerical stability. We also exploit low precision arithmetic to accelerate the iterations. Both the theoretical analysis and numerical experiments confirm the effectiveness of the resulting solver. Their performance is demonstrated through numerical experiments on an NVIDIA GPU using matrices arising from real-world applications.

Acknowledgements.

This research was supported by the Science Challenge Project under grant TZ2025007.

References

- [1] Björck A. (1967) Solving Linear Least Squares Problems by Gram–Schmidt Orthogonalization. *BIT*, Vol. 7, pp. 1–21.
- [2] Graillat S., Jézéquel F., Mary T., Molina R. (2024) Adaptive Precision Sparse Matrix-Vector Product and Its Application to Krylov Solvers. *SIAM J. Sci. Comput.*, Vol. 46, pp. C30–C56.
- [3] Zou Q. (2025) Rounding Error Analysis of the Inverse Compact WY Modified Gram–Schmidt Algorithms. *SIAM J. Matrix Anal. Appl.*, Vol. 46, pp. 726–747.

A Causal Discovery Framework for Interpretable Analysis of Digital Pathology in Breast Cancer

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Keywords. Digital pathology; causal discovery; breast cancer.

Artificial intelligence and computational methods are transforming oncology by enabling mechanistic insights from high-dimensional data. We present a causal discovery pipeline for digital pathology using the Multivariate Information-based Inductive Causation (MIIC) algorithm [1], which reconstructs probabilistic networks by identifying direct, indirect, and confounded dependencies while accounting for latent factors.

As a case study, 117 breast cancer patients undergoing neoadjuvant chemotherapy were analyzed. Pre-treatment H&E slides were processed to detect and classify tumor cells and tumor-infiltrating lymphocytes (TILs), extracting single-cell morphological, staining, and spatial features. Applying MIIC to final 46 integrated variables revealed causal relationships with pathological complete response. Key determinants included molecular subtype, nuclear morphology, and tumor-TIL spatial interactions. The network’s structure and dependencies were validated by expert pathologists, illustrating how probabilistic methods link quantitative tissue features to clinically relevant outcomes.

Acknowledgements.

This research was supported by Ricerca Corrente 2026 funded by the Italian Ministry of Health.

References

- [1] Sella N., Verny L., Uguzzoni G., et al. (2018) MIIC online: a web server to reconstruct causal or non-causal networks from non-perturbative data. *Bioinformatics*, vol. 34(13), pp. 2311–2313. DOI: 10.1093/bioinformatics/btx844

A Multimodal Learning Framework for Alzheimer’s Disease Characterization

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Keywords. Alzheimer’s disease; multimodal learning; data fusion; graph neural networks; predictive analysis.

Alzheimer’s disease analysis requires the integration of heterogeneous and high-dimensional data, including structural and functional neuroimaging, cognitive assessments, fluid biomarkers, genetic profiles, and demographic or clinical variables. The joint exploitation of these modalities is challenged by nonlinear interactions, incomplete observations, measurement noise, and the need to capture both individual and population-level disease patterns.

We propose a multimodal framework for Alzheimer’s disease modeling based on latent representation learning, attention-based fusion, and graph neural networks. Each modality is processed through a dedicated neural encoder, including autoencoder-based components, in order to extract compact and informative embeddings. The modality-specific embeddings are then integrated through an attention-based fusion mechanism that adaptively weights each source according to its predictive relevance. This yields a flexible representation able to preserve complementary multimodal information while mitigating modality imbalance.

To model relational dependencies among subjects or among brain regions, the framework incorporates graph neural networks with graph convolution operators defined on similarity graphs derived from phenotypic variables, imaging descriptors, or connectivity information. In this way, the architecture exploits topological structure through graph-based message passing, enhancing the representation of disease-related patterns beyond independent sample-wise prediction. From a computational perspective, the framework naturally lends itself to high-performance implementations, since multimodal encoding, attention-based fusion, and graph convolution operations can be parallelized and coupled with scalable optimization strategies for large-scale learning on heterogeneous biomedical data.

The proposed framework supports clinically relevant tasks such as the discrimination among cognitively normal subjects, patients with mild cognitive impairment, and patients with Alzheimer’s disease, as well as progression prediction in longitudinal settings. Beyond predictive accuracy, the learned latent variables, attention weights, and graph structures provide interpretable indicators of modality relevance and disease organization.

Approximation by Neural Networks via Time-Frequency Analysis

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Keywords. Approximation rate; neural network; modulation spaces; short-time Fourier transform; Barron space; curse of dimensionality.

We develop a quantitative approximation theory for shallow neural networks using tools from time-frequency analysis. Working in weighted modulation spaces $M_m^{p,q}(\mathbb{R}^d)$, we establish dimension-independent approximation rates in Sobolev norms $W^{n,r}(\Omega)$ for networks whose units combine standard activation functions with localized time-frequency windows.

Our main result shows that for $f \in M_m^{p,q}(\mathbb{R}^d)$, one can construct approximants f_N satisfying

$$\|f - f_N\|_{W^{n,r}(\Omega)} \lesssim N^{-1/2} \|f\|_{M_m^{p,q}(\mathbb{R}^d)},$$

on bounded domains $\Omega \subset \mathbb{R}^d$, with explicit control of all constants.

We also prove global approximation theorems on $\mathbb{R}^d$ using weighted modulation dictionaries, and derive consequences for Feichtinger's algebra, Fourier-Lebesgue spaces, and Barron spaces. Numerical experiments in one and two dimensions confirm that modulation-based networks achieve significantly improved Sobolev approximation compared to standard ReLU networks, in agreement with the theoretical estimates.

Acknowledgements.

This is joint work with Ahmed Abdeljawad (RICAM, Linz, Austria).

Fast and Interpretable Surrogate Models for Urban Flood Hazard Estimation

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Keywords. Urban flooding; surrogate modeling; machine learning; XAI.

We present an AI-based framework supporting fast and effective analyses of pluvial flood hazards, and its application to the urban area of Crotona, Italy. Extensive physics-aware simulations (performed by running a hydrodynamic model [1] over a HPC platform) are used to build a tabular view of various flood events where each row stores several features of a distinct spatial cell and of the event (including topography and forcing variables) and the final outcome of the simulation (in terms of water depth and velocity). While simulations offer reliable high-resolution estimates, their computational cost hinders operational use in various decision-making scenarios. We thus train surrogate AI models to approximate the simulation process and enable near-real-time forecasts. In concrete operational settings, however, predictive accuracy alone is not enough [3], since the predictions must be transparent enough to allow stakeholders assess their reliability [2]. We then devised the framework to allow for evaluating the accuracy–interpretability trade-off of surrogate models, using AI models with different levels of transparency in the case study. Specifically, we selected *Feed-forward Neural Network* (DNN) and *Random Forest* (RF) models as accurate but opaque surrogates, while using *Linear Tree Regressor* (LTR) models (i. e., decision trees with linear regressors in their leaves) and a *Linear Regressor* (LR) baseline as intrinsically-interpretable ones. Test results confirm that DNNs and RF models lead in accuracy, but the LTR one (which neatly outperforms the LR baseline) manages to provide quite accurate, and fully interpretable, predictions.

References

- [1] Costabile P. et al. (2026) Predicting pluvial flood impacts in data-scarce urban environments: Uncertainty and interplay between rainfall inputs and conceptual drainage loss models. *Urban Climate*, vol. 65, 102724.
- [2] Xing Z. et al. (2025) Fine-grained analysis and mapping of urban flood susceptibility with interpretable machine learning: A case study of Hefei, China. *Journal of Hydrology: Regional Studies*, vol. 60, 102501.
- [3] Cuzzocrea A. et al. (2025) Toward trustworthy and sustainable clinical decision support by training ensembles of specialized logistic regressors. *Neural Computing and Applications*, vol. 37, pp. 18233–18274.

Exploring Naïve and Tiled GPU Parallelization Strategies in Etnean Lava Flow Simulation

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Keywords. GPGPU; tiling; cellular automata; lava flow simulation.

Sciara-fv2 [4] is a Cellular Automata model for lava flow simulation, widely applied to Mt Etna, Italy (see, e.g., [1], and references therein). Differently from most stencil codes, the physics of lava flows is modeled according to [3], which makes cell's transition quite complex. This complexity poses an efficiency challenge to the GPU parallelization of the model, in consideration of the limited hardware resources of current devices. Nevertheless, a higher computational efficiency allows to study wider areas with a finer resolution, and in a shorter time. For this reason, alternative GPU parallelization versions of the model are here explored, from naïve [2] based on GPU's global memory, to tiled releases (with and without halo cells) which exploit shared memory. Results point out critical aspects of GPU parallelization of Sciara-fv2, which are common to other models based on [3], highlighting the need of innovative parallelization strategies.

References

- [1] Crisci G. M., Iovine G., Di Gregorio S., Lupiano V. (2008) Lava-flow hazard on the SE flank of Mt. Etna (Southern Italy). *Journal of Volcanology and Geothermal Research*, vol. 177(4), pp. 778–796.
- [2] D'Ambrosio D., Filippone G., Marocco D., Rongo R., Spataro W. (2013) Efficient application of GPGPU for lava flow hazard mapping. *Journal of Supercomputing*, vol. 65(2), pp. 630–644.
- [3] Di Gregorio S., Serra R. (1999) An empirical method for modelling and simulating some complex macroscopic phenomena by cellular automata. *Future Generation Computer Systems*, vol. 16, pp. 259–271.
- [4] Spataro W., Avolio M. V., Lupiano V., Trunfio G. A., Rocco R., D'Ambrosio D. (2010) The latest release of the lava flows simulation model SCIARA: first application to Mt Etna (Italy) and solution of the anisotropic flow direction problem on an ideal surface. *Procedia Computer Science*, vol 1(1), pp. 17–26. DOI: 10.1016/j.procs.2010.04.004

Hierarchy of Growth: From Ramsey Theory to Goodstein Sequences

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Keywords. Ramsey theory; unimaginable numbers; Goodstein sequences.

This talk explores the mathematical frontier where finite structures defy human intuition, focusing on the construction and behavior of unimaginable numbers. We begin with Ramsey Theory, demonstrating how unavoidable order emerges within large-scale chaos, and how this necessity leads to constants like Graham's Number, which transcend physical representation. To navigate these magnitudes, we analyze Knuth's Up-Arrow Notation and its recursive foundation, investigating the "density" of these operations and their role in the Fast-Growing Hierarchy.

The discussion then shifts to the fascinating dynamics of Goodstein Sequences. These sequences, while appearing to diverge toward infinity, eventually terminate at zero—a result that, remarkably, is unprovable within Peano Arithmetic. By examining the transition between different number bases and the role of transfinite ordinals (ϵ_0), we bridge the gap between elementary number theory and formal logic. Attendees will be guided through a journey across the "no-man's-land" between the computable and the infinite, reflecting on how these iterative models challenge our cognitive limits and redefine the boundaries of mathematical proof and visualization.

References

- [1] Caldarola F., d'Atri G., Maiolo M. (2021) What are the “unimaginable numbers”? In: Caldarola, F., d'Atri, G., Maiolo, M., Pirillo, G. (eds.), *From Pythagoras to Schützenberger. Unimaginable numbers*, Pellegrini Editore, pp. 17-29.
- [2] Caldarola F., d'Atri G., Mercuri P., Talamanca V. (2020) On the arithmetic of Knuth's powers and some computational results about their density. In: Sergeyev Y.D., Kvasov D.E. (eds.), *Numerical Computations: Theory and Algorithms 3rd Int. Conf.*, Springer, LNCS Vol. 11973, pp. 381–388.
- [3] Leonardis A., d'Atri G., Caldarola F. (2022) Beyond Knuth's notation for unimaginable numbers within computational number theory. *International Electronic Journal of Algebra*, vol. 31, pp. 55–73.

Quantum Computing for Molecules with Increasing Numbers of Electrons

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Keywords. SCF quantum algorithm; computational chemistry; Hartree–Fock; quantum computing; benchmarking.

Predicting the properties and reactivity of macromolecules and supramolecular aggregates of biological, pharmacological, and technological interest represents a fundamental challenge in computational chemistry. Despite the development of increasingly efficient algorithms, the computation of systems with a large number of electrons still requires significant computational time, even when using high-performance computing (HPC) systems. This limitation affects both the discovery of new molecular systems and the understanding of complex biochemical mechanisms. Quantum computing, characterized by an increasing number of qubits and progressively reduced error rates, represents a promising paradigm for overcoming these limitations [2]. In particular, quantum algorithms capable of computing molecular properties in reduced time compared to classical HPC approaches are expected to play a key role in the future of computational chemistry.

In this work, a Self-Consistent Field (SCF) procedure within the Hartree–Fock approximation [1] is designed to exploit quantum computational capabilities. The proposed approach is applied to molecular systems with increasing numbers of electrons and different geometries, including diatomic molecules and systems with steric numbers ranging from 2 to 6. Ground-state energies obtained through the SCF quantum algorithm are compared with those computed using classical methods. Although the proposed SCF quantum procedure is based on the Hartree–Fock approximation, representing a simplified framework for large systems, this study provides a first step toward understanding both the potential and the current limitations of quantum computing in molecular simulations.

References

- [1] Szabo A., Ostlund N. (2000) *Modern Quantum Chemistry: Introduction to Advanced Electronic Structure Theory*. Dover Publications, New York.
- [2] McArdle S., Endo S., Aspuru-Guzik A., Benjamin S. C., Yuan X. (2020) Quantum computational chemistry. *Reviews of Modern Physics*, vol. 92(1), 015003.

Beyond Euclidean Shorelines: Geometric Optimization of Oscillating Water Columns for Coastal Defense

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Keywords. Oscillating water column (OWC); sustainable energy production; coastal mathematical models.

This talk explores the physical-mathematical modeling and numerical simulation of Oscillating Water Column (OWC) systems, focusing on their integration into coastal defense infrastructures. Building upon the research published in [1], we analyze the hydrodynamic performance of OWC chambers with varying trapezoidal cross-sections. A primary challenge lies in optimizing the front wall inclination (from 90° to 45°), where an optimal efficiency of approximately 12.8% is identified at 55° as a balance between internal compression and wave reflection interference.

To bridge the gap between energy harvesting and environmental integration, the study incorporates an innovative geometric approach: the use of iterative functional systems and fractal modeling to characterize the irregular morphology of coastal boundaries. By treating the shoreline not as a smooth Euclidean line but as a complex iterative curve, the model provides a more robust framework for assessing hydraulic loads and wave-energy dissipation. This dual approach—combining structural OWC optimization with fractal coastal modeling—transforms traditional breakwaters into deterministic engineering tools. The results offer significant structural and construction benefits, providing a comprehensive solution for sustainable energy production and high-precision coastal protection in the face of evolving maritime dynamics.

References

- [1] Caldarola F., Carini M., Costarella A., De Raffeale G., Maiolo M. (2025) Physical-Mathematical Modeling and Simulations for a Feasible Oscillating Water Column Plant. *Mathematics*, Vol. 13, 2019.
- [2] Caldarola F., Carini M., Maiolo M., Papaleo M. A (2024) Iterative Mathematical Models Based on Curves and Applications to Coastal Profiles. *Mediterranea Journal of Mathematics*, Vol. 21, 178.
- [3] Caldarola F., Maiolo M. (2020) On the Topological Convergence of Multi-rule Sequences of Sets and Fractal Patterns. *Soft Computing*, Vol. 24(23), pp. 17737–17749.

Community Detection in Graphs: From Classical Algorithms to Graph Neural Networks

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Keywords. Community detection; graph partitioning; graph neural networks.

Community detection in graphs is a fundamental problem that has been extensively studied and applied to a wide range of real-world networks. As the size and complexity of modern graph data have increased, new methodological approaches have been developed to address the limitations of classical techniques. In particular, methods based on neural networks and deep learning have recently gained significant attention, as they are better suited to handle large-scale and complex graph structures. In this talk, we first introduce the main concepts underlying Graph Neural Networks (GNNs), presenting a taxonomy of existing models commonly adopted in the literature. We then review the principal approaches to graph partitioning and community detection, starting from classical algorithms and gradually moving toward recent deep learning-based methods, highlighting their main ideas, advantages, and limitations.

Acknowledgements.

This research was supported by the Grant For Internationalization (GFI) 2025 Project funded by the University of Turin.

References

- [1] Cavoretto R., De Rossi A., Lancellotti S., Romaniello F. (2024) Node-bound communities for partition of unity interpolation on graphs. *Appl. Math. Comput.*, vol. 467, pp. 231–237.
- [2] Cavoretto R., Comoglio C., De Rossi A. (2026) Community detection methods for GBF-PUM signal approximation on graphs. *Appl. Math. Comput.*, vol. 510, 129702.

New Families of Nonconforming Histopolation Methods for Function Reconstruction On Polygonal Meshes

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Keywords. Reconstruction of functions; nonconforming weighed finite element; exponential Gegenbauer weight.

We address the construction of advanced nonconforming finite element schemes on general polygonal meshes, motivated by the increasing need for high-order accurate methods in complex geometries and data reconstruction problems. In this context, we introduce new families of nonconforming higher-order Crouzeix–Raviart type finite elements specifically designed for function reconstruction from integral data. The proposed framework relies on degrees of freedom defined through weighted edge moments of orthogonal polynomials, which guarantee the exact reproduction of all polynomials up to degree m on each element. This formulation represents a bridge between the finite element methods and the orthogonal polynomial theory. A complete unisolvence theory is established, revealing that the solvability of the local approximation problem depends on the parity of the product mN , where m is the polynomial degree and N the number of polygon edges. When this condition fails, an enrichment strategy based on an additional functional and a suitably constructed enrichment function restores unisolvence and stability. The resulting enriched Crouzeix–Raviart type elements provide a systematic and unified framework for higher-order nonconforming approximations on arbitrary polygonal meshes. Comprehensive numerical experiments demonstrate the high accuracy, robustness, and flexibility of the proposed elements across different polynomial orders and mesh configurations.

Acknowledgements. Travel expenses partially covered by a travel award sponsored by the open access journal Mathematics published by MDPI.

References

- [1] Dell’Accio F., Guessab A., Milovanović G. V., Nudo F. (2026) Nonconforming approximation methods for function reconstruction on general polygonal meshes via orthogonal polynomials, *IMA J. Numer. Anal.*, accepted.

Asymptotic Properties of Geometric Multidimensional Scaling

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Keywords. Optimization; geometric multidimensional scaling; asymptotics.

Multidimensional scaling (MDS) is a classic approach for dimensionality reduction of multidimensional data. It is a complex global optimization problem. Geometric Multidimensional Scaling (GMDS) is an iterative method that ensures analytically defined, geometrically grounded minimization of the stress. The ideas of GMDS were presented at NUMTA-2019 [1]. A complete theoretical foundation is given in [2]. GMDS is a generalization of the SMACOF method [3]. Our research provides an asymptotic analysis of GMDS. Let us have m objects $X_1, \dots, X_m$ characterized by a dissimilarity matrix. MDS aims to find an optimal configuration of m points $Y_1, \dots, Y_m$, $Y_i = (y_{i1}, y_{i2}, \dots, y_{id})$. If $d = 2$, we get a mapping of the objects on a plane. GMDS minimizes the raw stress by updating $Y_1, \dots, Y_m$ at each iteration. At each step, GMDS finds a new position Y_j^* corresponding to X_j , starting from the current point Y_j . In this research, we examine a situation in which we freely vary one point, Y_j , while keeping the other points fixed. When the point Y_j is chosen infinitesimally close to a fixed point Y_k ($k \neq j$), the set of possible resulting positions Y_j^* after the geometric step from Y_j forms a d -dimensional hypersphere with a known center and radius. When we move Y_j to infinity, the set of possible resulting positions Y_j^* forms a hypersphere whose center is the mean point of the remaining points $Y_i, i \neq j$, and whose radius is the average dissimilarity of X_j with all $X_i, i \neq j$. The theoretical findings allow a better understanding of GMDS behavior and will be useful for improving its implementation for global optimization of the stress function.

References

- [1] Dzemyda G., Sabaliauskas M. (2020) A novel geometric approach to the problem of multidimensional scaling. In *Numerical Computations: Theory and Algorithms. NUMTA 2019. Springer Series Lecture Notes in Computer Science*, Vol. 11974 (ed. by Sergeyev, Y.D., Kvasov, D.E.), Springer, Cham, pp. 354–361.
- [2] Dzemyda G., Sabaliauskas M. (2024) Geometric multidimensional scaling: Efficient approach for data dimensionality reduction. *Journal of Global Optimization*, Vol. 88, pp. 53–77.
- [3] De Leeuw, J., Mair, P. (2009) Multidimensional scaling using majorization: SMACOF in R. *Journal of Statistical Software*, Vol. 31, No. 3, pp. 1–30.

Sub-pixel estimation of the shift between two one-dimensional signals.

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Keywords. Sub-pixel accuracy; motion estimation; Fourier transforms.

We propose a method based on the Discrete Fourier Transform (DFT) for evaluating the magnitude of the shift between two consecutive signals. The proposed method has been obtained by working in the Fourier domain and leveraging the corresponding well-known Fourier Shifting Property. The method has a sub-pixel accuracy and has been compared with other methods based on the classical estimation of the phase-correlation maxima [1]. This work uses results [2] obtained within the framework of Motion Magnification (MM), which is a recent subject in the Image Processing having various applications in engineering and medicine. The techniques developed within MM aim to support the human visual system in order to capture small modifications in colour and/or position.

References

- [1] Alba A., Viguera-Gomez J.F., Arce-Santana E.R., Aguilar-Ponce R. M. (2015) Phase correlation with sub-pixel accuracy: A comparative study in 1D and 2D. *Computer Vision and Image Understanding*, vol. 137, pp. 76–87.
- [2] Egidi N., Giacomini J., Leonesi P., Maponi P., Mearelli F., E. Trebović (2025) Image Processing for Motion Magnification. *Journal of Approximation Software*, vol. 2, p. 2.

Sparse Data Representation with Graph Wedgelet Decompositions

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Keywords. Graph wedgelets; data compression; adaptive wedge partitions.

Graph wedgelets [1] provide a graph-based framework for the efficient compression and representation of data. It is based on the adaptive construction of a binary graph partitioning tree tailored to the structure of a graph signal, generating a highly sparse and memory-efficient signal representation. The adaptivity of the partitioning is achieved through recursive wedge splits recursively subdividing the regions of the graph in which the representation error is large.

In this study, we focus on refinements of this construction to increase the sparsity of signal representations. For this, we consider extended families of basis functions able to approximate graph signals locally with higher accuracy. By increasing this approximation quality, less recursive wedge splits are required leading to more compact partitioning trees. In addition, we investigate the role of adaptivity in this scheme. While adaptivity is important for sparse representations, it also leads to additional computational overhead. For this, we explore strategies to moderate the degree of adaptivity while preserving approximation quality to reduce both, computational complexity and storage costs. Finally, we show that this method is also practical for real-world applications and can be used for the compression of traditional images, as well as inside a split-and-merge segmentation scheme for biomedical data [2].

Funding by the University of Padova, Department of Mathematics, project SID BIRD 2023, “AL-ISIA - ALgorithms for Immersive Stereoscopic Imaging with Applications to the Daedalus camera system”, by the European Union – NextGenerationEU under the National Recovery and Resilience Plan (NRRP), Mission 4 Component 2 Investment 1.1 - Call PRIN 2022 No. 104 of February 2, 2022 of Italian Ministry of University and Research; Project 2022FHCNY3 (subject area: PE - Physical Sciences and Engineering) “Computational mEthods for Medical Imaging (CEMI)”, and by the INdAM research group GNCS (project CUP_E53C24001950001) **is acknowledged**.

References

- [1] Erb W. (2023) Graph Wedgelets: Adaptive Data Compression on Graphs based on Binary Wedge Partitioning Trees and Geometric Wavelets. *IEEE Trans. Signal Inf. Process. Netw.*, vol. 9, pp. 24–34.
- [2] Erb W. (2026) Split-and-Merge Segmentation of Biomedical Images Using Graph Wedgelet Decompositions. In: *Computational Science and Its Applications – ICCSA 2025* (Gervasi O. et al., eds). LNCS, vol. 15899, Springer, pp. 252–263.

On the Study of Chattering-Like Phenomena in k -Order State-Constrained Control Problems

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Keywords. Optimal control; state constraints; maximum principle.

The work is aimed at the study of one class of linear k -order state-constrained optimal control problems. In our formulation, the control is the k -order derivative of the state position, which is supposed scalar. The constraints of the problem are given by bounds on the control and state position, while the k -valued starting and terminal points are assumed to be fixed. One interesting property of this problem is the effect of contact-points accumulation demonstrating a chattering-like behaviour of the extremal trajectory relatively the boundary of the set given by the state constraint. The number of contact-points is predetermined by the boundary conditions. At the same time, the cost can change, which means that it is possible to point out to an entire stock of problems satisfying the chattering property, since the constraint-order is of the most importance here.

This paper expands on the research efforts undertaken in [1] and explores this problem at a new level, by examining higher order constraints and a larger number of contact-points. A few cases with different costs are also considered. Some theoretical analysis based on the Pontryagin maximum principle is carried out and a computational algorithm is proposed to solve the problem numerically for a large number of contact-points. We also compare the extremals computed within the indirect method with the numerical solution obtained by the direct method based on optimization package IPOPT [2].

References

- [1] Zhukova A. A., Karamzin D. Y. (2025) Calculation of Extremals in an Optimal Control Problem with a Higher-Order State Constraint. *Comput. Math. and Math. Phys.*, vol. 65, pp. 2838–2853.
- [2] Wächter A., Biegler L. T. (2006) On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Math. Program.*, vol. 106(1), pp. 25–57.

A Parallel On-Premise Multimodal RAG for a Privacy-Preserving Knowledge Management

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Keywords. Multimodal retrieval-augmented generation; parallel computing; on-premise AI; privacy-preserving knowledge management.

Deploying AI-based knowledge retrieval systems in high-security sectors, such as healthcare, civil protection, and defense, requires the design of on-premise architectures capable of meeting rigorous real-time operational constraints while ensuring that sensitive data remains localized without leaking to any external services [1]. Within these critical domains of operation, a sequential AI pipeline often fails to satisfy the requirements of handling large-scale and heterogeneous documents. In order to overcome performance bottlenecks and guarantee scalability, the architectural paradigm has to shift towards parallel architectures.

To address these challenges, this work introduces a computational framework for on-premise Multimodal Retrieval-Augmented Generation (MRAG) centered on a parallel execution strategy [2]. The system integrates multimodal document processing, dual-modality semantic embedding for both text and images, vector-based similarity search, evidence re-ranking, and local generative inference. The framework targets three main computational bottlenecks through parallelization: (i) embedding generation over large multimodal corpora, achieved via data-parallel distribution across multiple processing units; (ii) vector indexing and approximate nearest-neighbor search, scaled across a multi-node distributed configuration; and (iii) pipeline-level inference latency, minimized through asynchronous producer-consumer scheduling between the retrieval, re-ranking, and generation stages.

Preliminary experiments validated the effectiveness of the proposed parallel strategy, confirming the practical applicability of the framework to real-world, privacy-constrained deployment scenarios.

References

- [1] Chirumamilla K. M., Siddique M. A. S. (2026) Privacy-Preserving RAG System for Personal Document Question Answering. In: *2026 IEEE 16th Annual Computing and Communication Workshop and Conference (CCWC)*, pp. 0204–0213. IEEE.
- [2] Şakar T., Emekci H. (2025) Maximizing RAG efficiency: A comparative analysis of RAG methods. *Natural Language Processing*, vol. 31, no. 1, pp. 1–25.

Towards Precision-Preserved Computations in Neural Networks with the Infinity Computer

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Keywords. Softmax; Cross-Entropy; Infinity Computer; Neural Networks; Numerical Stability.

Softmax activation and Cross-Entropy loss are widely used in the training of artificial neural networks, particularly in multi-class classification tasks. When implemented on conventional digital computers, these functions may suffer from numerical instability in the presence of very large or very small input values. In particular, exponentiation and logarithmic operations involved in their computation may cause overflow, underflow, or loss of precision due to the limitations of floating-point arithmetic.

This work investigates an alternative computational approach aimed at improving the numerical robustness of neural network computations by exploiting the Infinity Computer paradigm. This framework introduces a numerical methodology allowing finite, infinite, and infinitesimal quantities to be represented and manipulated within the same positional numeral system.

Within this framework, precision-preserved formulations of neural network computations are proposed. The approach introduces infinitesimal quantities acting as numerical safeguards during the computation process. Numerical experiments performed using a Python implementation based on the Py-Grossone library show that the proposed approach behaves consistently in regular scenarios and remains stable in situations where standard floating-point implementations may produce undefined values. These results indicate that computational paradigms capable of handling infinitesimal quantities can improve the robustness of numerical procedures in machine learning.

References

- [1] Sergeyev Y. D. (2017) Numerical infinities and infinitesimals: methodology, applications, and repercussions on two Hilbert problems. *EMS Surveys in Mathematical Sciences*, vol. 4(2), pp. 219–320.
- [2] LeCun Y., Bengio Y., Hinton G. (2015) Deep learning. *Nature*, vol. 521, pp. 436–444.

Parametric Surface Modelling via ANFIS and B-spline Quasi-Interpolation

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Keywords. Surface modelling; spline quasi-interpolation; neural networks.

In CAD systems, a standard representation of a computational domain Ω is usually provided in terms of its boundary, but in order to perform numerical simulation, a representation of the interior is often necessary, i.e., a parametric surface model is therefore constructed. More rigorously, a bijective, and, hence, a folding-free mapping $\mathcal{M}$, defined on a reference domain $\hat{\Omega} = [0, 1]^2$, is computed such that:

$$\mathcal{M} : \hat{\Omega} \rightarrow \Omega$$
$$\forall (t_1, t_2) \in \hat{\Omega}, \exists (\mathbf{x}, \mathbf{y}) \in \Omega : \mathcal{M}(t_1, t_2) = (\mathbf{x}(t_1, t_2), \mathbf{y}(t_1, t_2)).$$

We propose to approximate the sought mapping $\mathcal{M}$ via an Adaptive Network Fuzzy Inference System (ANFIS) [1]. In our architecture, the fuzzy inference *if-then rules* are Sugeno-kind with Gaussian type membership functions. Fuzzy inference systems are based on fuzzy theory and sets. Additionally, they can emulate human like reasoning in a precise manner because the inference rules are pre-defined and embedded in the FIS. They are computationally efficient and guarantee the continuity of the output surface [2], making them well-suited for mathematical analysis. In the proposed method, the final continuous parameterization is expressed in terms of B-splines and it is obtained by means of the Hermite type quasi-interpolation [3].

Acknowledgements.

This research was supported by the INdAM GNCS Projects.

References

- [1] Jang J. S. (1993) ANFIS: adaptive-network-based fuzzy inference system. *IEEE transactions on systems, man, and cybernetics*, vol. 23, pp. 665–685.
- [2] Mehran K. (2008) Takagi-sugeno fuzzy modeling for process control. *Industrial Automation, Robotics and Artificial Intelligence* (EEE8005), vol. 262, pp. 1–31.
- [3] Bertolazzi E., Falini A., Mazzia F. (2022) The object oriented C++ library QIBSH++ for Hermite spline quasi interpolation. ArXiv preprint [arXiv:2208.03260](https://arxiv.org/abs/2208.03260).

Handling Degeneracy in Conjugate Gradient (CG) and Conjugate Residual (CR) Methods

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Keywords. CG method; CR method; Grossone.

Iterative Krylov subspace methods such as the Conjugate Gradient (CG) and Conjugate Residual (CR) algorithms are widely employed for solving large symmetric linear systems arising in scientific computing and optimization. However, when the system matrix is indefinite, these methods may encounter numerical breakdowns or severe instabilities, typically associated with vanishing denominators or loss of orthogonality. In particular, degeneracy can arise when quantities such as $v^T A v$ approach zero, preventing the computation of a valid step length and interrupting the iteration process.

This work on one hand investigates the behavior of CG and CR methods in the presence of an indefinite matrix. Then, it explores strategies to maintain algorithmic robustness under possible breakdown. We analyze the algebraic structure underlying breakdown situations and discuss their interpretation within the framework of quadratic functionals associated with the linear system.

A central component of the investigation is the use of the numeral system based on the infinite unit grossone, which allows the explicit numerical representation of infinitesimal and infinite quantities within a consistent computational framework.

Acknowledgements. This research was supported by: National Research Council–Marine Technology Research Institute (CNR–INSEAN), Italy; GNCS of INdAM (Istituto Nazionale di Alta Matematica), Italy.

References

- [1] De Leone R., Fasano G., Sergeyev Y.D. (2018) Planar methods and Grossone for the Conjugate Gradient breakdown in Nonlinear Programming. *Computational Optimization and Applications*, vol. 71, pp. 73–93.
- [2] Dahito M.-A., Orban D. (2019) The Conjugate Residual Method in Linesearch and Trust-Region Methods. *SIAM Journal on Optimization*, vol. 29(3), pp. 1988–2025.
- [3] Luenberger D.G. (1970) The Conjugate Residual Method for Constrained Minimization Problems. *SIAM Journal on Numerical Analysis*, vol. 7(3), pp. 390–398.

Nonlinear Diffusion Methods for the Processing of 2D and 3D Images

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Keywords. Nonlinear diffusion; image denoising; image deblurring.

We discuss the image processing methodology introduced in [1] in the context of nonlinear diffusive PDEs. The main theoretical and numerical results developed so far concern the case of two-dimensional images, where a corrupted image u is represented in a continuous framework and used as initial condition for the evolution equation

$$\frac{\partial u}{\partial t} = \sqrt{\frac{|\nabla u|^2}{1 + |\nabla u|^2}} \Delta u,$$

supplemented with homogeneous Neumann boundary conditions. Contrary to classical approaches, such as those stemming from the pioneering work of Perona and Malik [2], the nonlinear diffusion coefficient depends locally on $|\nabla u|$ and grows proportionally to this quantity. An ad-hoc discretization scheme, based on centered finite differences on two interlaced grids and on a backward Euler time discretization, has been proved effective in the two-dimensional setting for denoising, deblurring, edge detection, and segmentation.

The main contribution of the present work concerns the extension of this methodology to the processing of three-dimensional images. The proposed numerical approach naturally generalizes to the three-dimensional setting and proves effective for the enhancement and restoration of 3D images affected by noise and blur. Applications to real 3D images are also presented.

References

- [1] Fatone L., Funaro D. (2022) High-order discretization of backward anisotropic diffusion and application to image processing. *Annali Università di Ferrara*, vol. 68(2), pp. 295–310.
- [2] Perona P., Malik J. (1990) Scale space and edge detection using anisotropic diffusion. *IEEE Trans. on Pattern Anal. and Machine Intelligence*, vol. 12(7), pp. 629–639.

Complex Transform Analysis of Differential Equations in Fractional Impulsive Systems

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Keywords. Impulsive fractional differential equations; analytical solutions; dynamical systems.

This study presents an analytical approach for solving fractional impulsive differential equations using the fractional complex transformation method. Impulsive models are employed instead of classical differential equations when instantaneous external interventions or internal disruptions occur within energy systems. Such models are particularly important in practical applications, as real-world energy systems rarely operate under perfectly smooth conditions. The main objective of the study is to simplify the solution procedure through the use of the fractional complex transform, which converts fractional derivatives into classical derivatives and thereby significantly reduces the computational complexity associated with nonlinear terms. By combining this transformation with analytical iterative methods, the effects of the fractional order and impulsive jumps are examined.

References

- [1] Li Z.B., He J.H. (2010) Fractional complex transform for fractional differential equations. *Mathematical and Computational Applications*, vol. 15, pp. 970–973.
- [2] Ma C. (2019) A novel computational technique for impulsive fractional differential equations. *Symmetry*, vol. 11, article 216. DOI: 10.3390/sym11020216.
- [3] Mahdy A.M.S., Marai G.M.A. (2018) Fractional complex transform for solving the fractional differential equations. *Global Journal of Pure and Applied Mathematics*, vol. 14, pp. 17–37.
- [4] Faydaoğlu Ş. (2026) Accurate analytical solutions of fractional nonlinear oscillator systems with matching approach. *Archive of Applied Mechanics*, vol. 96, article 39. DOI: 10.1007/s00419-026-03030-7.

Numerical techniques for multispectral image demosaicing

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Keywords. Image demosaicing; quasi-interpolation; tensor decomposition.

Digital color cameras capture only three broad spectral bands, losing most of the available spectral information. Multispectral imaging addresses this limitation, and among snapshot single-sensor systems we focus on Multispectral Filter Arrays (MSFAs). Recovering the full spectral definition— a problem known as *demosaicing*—requires estimating missing channels. Classical methods for non-redundant MSFAs include weighted bilinear, intensity difference [1], and scattered data interpolation, all of which are synthesized using tensor-decomposition techniques such as Tensor-Train and Tucker [2].

This work introduces for the first time within the demosaicing context the BSHermite Quasi-Interpolation (QI) scheme. In particular, we employ a variant of the QI operator, introduced in [3], which uses finite-difference formulas to approximate the unknown derivatives at the spline knots—that is, at the known pixel locations. Furthermore, thanks to its optimal approximation-power properties, the QI operator ensures matching interfaces with minimal error [4].

These approaches are evaluated on a simulated 9-band Earth Observation dataset, designed to reflect the requirements of the ESA-InCubed AIX mission.

Acknowledgements.

This study has been conducted in collaboration with Planetek Italia S.r.l.

References

- [1] Mihoubi S., Losson O., Mathon B., Macaire L. (2015) Multispectral demosaicing using intensity-based spectral correlation. In: *2015 International Conference on Image Processing Theory, Tools and Applications (IPTA)*, IEEE, pp. 461–466.
- [2] Ballard G., Kolda T. G. (2025) *Tensor Decompositions for Data Science*, Cambridge University Press.
- [3] Mazzia F., Sestini A. (2009) The BS class of Hermite spline quasi-interpolants on nonuniform knot distributions. *BIT Numerical Mathematics*, vol. 49, pp. 611–628.
- [4] Bertolazzi E., Falini A., Mazzia F. (2022) The Object Oriented C++ library QIBSH++ for Hermite spline Quasi Interpolation. *ArXiv*: 2208.03260.

Structure-Preserving Orthogonal Integrators for Isospectral Flows

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Keywords. Matrix differential equations; multistep integrators; orthogonal projections.

Isospectral flows arise in several applications and are governed by matrix differential equations of the form $L' = BL - LB$, where L is symmetric and B is skew-symmetric. The exact solution preserves important properties, including the symmetry and the spectrum of L . Numerical integrators that maintain these properties are therefore of particular interest.

In this work we investigate orthogonal integrators for isospectral flows and consider two classes of structure-preserving schemes: automatic orthogonal integrators and projected orthogonal integrators. For the latter, we study projection techniques based on the QR and polar decompositions and examine their numerical behaviour in the context of isospectral flows.

Since the computation of higher-order derivatives required by Taylor methods becomes prohibitively expensive for large systems, we focus on multistep and predictor-corrector approaches. Finally, we compare the proposed approaches with Lie group methods commonly used for matrix differential equations and discuss their behaviour with respect to the preservation of the underlying geometric structure.

Acknowledgements.

This research was supported by the National Natural Science Foundation of China under Grant No. 11371075 and by Portuguese funds through FCT – Fundacao para a Ciencia e a Tecnologia, within Project UID/00013/2025: Centro de Matematica da Universidade do Minho (CMAT/UM).

References

- [1] Higham D. J. (1997) Time-stepping and preserving orthonormality. *BIT*, vol. 37(1), pp. 24–36.
- [2] Iserles A., Munthe-Kaas H. Z., Nørsett S. P., Zanna A. (2005) Lie-group methods. *Acta Numerica*, vol. 9(1), pp. 1–148.
- [3] Kaur A. (2016) On solving an isospectral flow. *J. Comp. Appl. Math.*, vol. 308, pp. 263–275.
- [4] Lopez L., Politi T. (1997) Numerical procedures based on Runge-Kutta methods for solving isospectral flows. *Appl. Numer. Math.*, vol. 25, pp. 443–459.

Artefact Synergy and AI-Enhanced Modelling: Advancing Proportional Reasoning in Integrated STEM Education

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Keywords. STEM education; AI tools; semiotic mediation.

Mathematical modelling [1] is increasingly recognised as a key practice in integrated STEM education. This study investigates how the synergy between manipulative, digital, and AI-supported tools can foster students' understanding of proportional relationships. The research is grounded in the Theory of Semiotic Mediation [1], which interprets artefacts as mediators supporting the transition from situated actions to shared mathematical meanings.

A teaching experiment conducted in an Italian lower secondary school engaged students with Brousseau's puzzle [3], first reconstructed through manual manipulation and then enlarged according to a proportional rule. Students subsequently modelled the puzzle in a digital 3D environment, where proportional relationships were expressed through scale factors. An additional AI-supported feedback phase was introduced during the validation stage of the modelling cycle, allowing students to verify proportional consistency and reflect on discrepancies between representations.

Preliminary observations suggest that the coordinated use of physical artefacts, digital modelling tools, and AI-mediated feedback supports students in articulating proportional reasoning and connecting embodied exploration with formal mathematical representations. The study highlights the potential of artefact synergy to sustain modelling processes and to link mathematical reasoning with technological design practices in STEM education.

References

- [1] Bartolini Bussi M. G., Mariotti M. A. (2008) Semiotic mediation in the mathematics classroom. In: *Handbook of International Research in Mathematics Education*, second revised edition (pp. 746–783). New York and London: Routledge (Taylor & Francis Group).
- [2] Blum W., Ferri R. B. (2009) Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application*, vol. 1 (1), pp. 45–58.
- [3] Brousseau G. (1997) *Theory of Didactical Situations in Mathematics*. Kluwer Academics.

Scaling Federated Graph Neural Networks via Quantised and Sparse Updates

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Keywords. Federated learning; graph neural networks.

Federated learning (FL) enables collaborative training across decentralised data silos without sharing raw data, making it attractive for privacy-sensitive graph-structured datasets. However, many federated graph neural network (GNN) frameworks still transmit full-precision model updates at every round, resulting in high communication overhead that limits scalability under bandwidth and latency constraints [1]. In this work, we study communication-efficient strategies for federated GNN training, focusing on compressed update schemes based on quantisation and sparsification, and discussing aggregation choices to better cope with client heterogeneity.

We identify key bottlenecks in existing federated GNN systems—such as full-model replication and limited compression support—and quantify the potential savings enabled by lightweight update transmission. We evaluate the impact of the proposed strategies on convergence behaviour, predictive performance, and transmitted bytes per round using real-world datasets related to coordinated information operations (IOs) across multiple geopolitical contexts [2]. The results indicate that appropriately calibrated compression can substantially reduce communication costs while maintaining competitive predictive performance, supporting practical deployment of federated GNNs in high-volume, privacy-constrained settings.

Acknowledgements.

This work was partially supported by the research project **SERICS** (PE000000014) under the MUR National Recovery and Resilience Plan funded by the European Union – NextGenerationEU. The support is gratefully acknowledged.

References

- [1] Folino G., Forestiero A., Papuzzo G., Spezzano G. (2010) A grid portal for solving geoscience problems using distributed knowledge discovery services. *Future Gener. Comput. Syst.* vol.26(1), 87–96.
- [2] Folino G., Minici M., Sabatino P., Tenghi M.S. (2026) An efficient and scalable federated graph neural network architecture for information operation detection. In: *34th Euromicro International Conference on Parallel, Distributed and Network-Based Processing, PDP 2026*, Cluj-Napoca, Romania, 25-27 March 2026. IEEE

Informed Vertical Federated Learning for Effective Cross-Organizational Cybersecurity

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Keywords. Federated learning; ensemble; cybersecurity.

The increasing sophistication and scale of cyber threats challenge traditional data-driven defence strategies based on a centralised training of Machine Learning models on historical data. In highly-regulated environments, sharing raw cybersecurity data across organisational boundaries is infeasible, calling for federated learning approaches that preserve privacy. A *Vertical Federated Learning* (VFL) scheme, where sensitive data features are only used locally in each organisation, at both training and application times, offers a promising solution for cross-organisational settings. However, since local models consider different subsets of the data features, even when combined via an ensemble scheme, this approach may yield substantially less accurate predictions than centralised learning. This risk can be mitigated by complementing the local models with a pre-trained one that captures and leverages complementary domain knowledge. In this work, we introduce a VFL approach, adopting a novel distributed MoE (Mixture of Experts) architecture, which allows for incorporating prior knowledge available as pre-trained expert models, in addition to a small number of shallow local experts for the sake of efficiency and explainability [1] [2]. Specifically, in the approach, prior expert models are trained on synthetic data –this training process, entailing no risk of sensitive data disclosure, could be done at the coordinator level, ignoring the vertical partitioning of the data. This prior knowledge is then distributed to each organisation (e.g., as model parameters). Experiments on a well-known real-life cybersecurity dataset confirm the validity of the proposed approach, by showing its ability to achieve a satisfactory balance between attack prediction accuracy and efficiency, while ensuring good levels of prediction explanation and of local information protection by design.

References

- [1] Folino F., Folino G., Pisani F.S., Pontieri L., Sabatino P. (2024) Efficiently approaching vertical federated learning by combining data reduction and conditional computation techniques. *J. Big Data*, vol. 11(1), article 77.
- [2] Folino F., Folino G., Pisani F.S., Sabatino P., Pontieri L. (2024) A scalable vertical federated learning framework for analytics in the cybersecurity domain. In: Chis A.E., González-Vélez H. (eds.) *32nd Euromicro Intl Conference on Parallel, Distributed and Network-Based Processing, PDP 2024*, Dublin, Ireland, March 20-22, 2024. IEEE, pp. 245–252.

Positivity-Preserving Methods for Time-Fractional RAD Equations

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Keywords. Nonstandard finite difference methods; time-fractional reaction-advection-diffusion equations; positivity-preserving numerical schemes.

Nonstandard finite difference (NSFD) methods, originally introduced by Ronald E. Mickens [1], represent a class of simple yet robust numerical schemes for the approximation of differential equations. A distinctive feature of these methods is their ability to preserve key qualitative properties of the underlying continuous models. In particular, NSFD schemes can guarantee the positivity of the numerical solution independently of the mesh size, providing a reliable and computationally efficient alternative to many classical discretization techniques, which typically maintain positivity only under restrictive step-size conditions.

Recently, several extensions of NSFD methods have been proposed for a broader range of functional equations, highlighting the flexibility of the NSFD framework in constructing structure-preserving schemes for complex systems.

In this talk, we focus on time-fractional reaction-advection-diffusion (RAD) problems, which arise in the modeling of anomalous transport phenomena in fields such as physics, chemical engineering, and material science. For such models, ensuring the positivity of the numerical solution is often essential from both physical and mathematical viewpoints, yet it may be difficult to achieve with standard discretization strategies.

We present the construction and theoretical analysis of some new NSFD schemes tailored to these time-fractional problems [2]. The proposed methods are designed to preserve positivity while maintaining computational efficiency. Their stability and qualitative properties are discussed, and numerical experiments are provided to illustrate the theoretical results and to compare the performance of the proposed approaches with that of classical numerical methods.

References

- [1] Mickens R. E. (1993) *Nonstandard Finite Difference Models of Differential Equations*. World Scientific.
- [2] Cardone A., Frasca-Caccia G., Paternoster B. (2026) Non-standard schemes for time-fractional reaction-advection-diffusion problems. *J. Comput. Appl. Math.*, vol. 471, article 116757.

Poisoning Attacks in TSVM Models

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Keywords. Adversarial machine learning; TSVM; nonsmooth optimization.

The increasing adoption of Machine Learning in security-critical domains has raised significant concerns about robustness against strategic adversaries. Adversarial Machine Learning [1] addresses this issue by modelling attacks as deliberate perturbations designed to alter the classifier’s decision boundary, thereby violating the classical independent and identically distributed assumption.

In semi-supervised setting [2], the Transductive Support Vector Machine (TSVM) approach extends the standard SVM model by incorporating unlabeled data and exploiting the global geometry of the dataset under the cluster assumption. While this typically improves generalization performance, it also introduces new vulnerabilities, as the decision boundary becomes sensitive to the overall data distribution.

In this work, we propose novel poisoning attack models specifically designed for TSVM. The attacker perturbs the unlabeled samples to induce controlled deformations of the separating hyperplane. This problem can be formulated as a bilevel optimization program, where the upper level maximizes an adversarial objective and the lower level corresponds to TSVM training. Both levels involve nonconvex and nonsmooth objective functions, whose minimization can be performed using DC (Difference-of-Convex) type algorithms.

After presenting preliminary numerical results, we also discuss possible strategies to mitigate such threats. In this perspective, robust TSVM training can be adopted to account for worst-case attacks, providing a safeguard defence mechanism and ensuring stability of the decision boundary.

References

- [1] Astorino A., Gaudioso M., Gorgone E., Manca B. (2025) Difference of Convex programming in adversarial SVM. *Journal of Computational and Applied Mathematics*, vol. 454, 116201.
- [2] Astorino A., Fuduli A. (2007) Nonsmooth optimization techniques for semisupervised classification. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 29, pp. 2135–2142.

Hybrid Deep Learning and Multiple Instance Learning for Diabetic Retinopathy Detection

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keywords. Multiple instance learning; diabetic retinopathy detection; ResNet50.

Diabetic Retinopathy (DR) is a severe complication of diabetes mellitus and the leading cause of preventable blindness in the working-age population, affecting approximately 30% of diabetic patients worldwide and projected to grow significantly in the coming decades. Automated and cost-effective screening tools are therefore of paramount importance.

In a previous work [1], we addressed the binary classification problem of discriminating eye fundus images of healthy individuals from those of patients with severe DR, using the **mi-SPSVM** (multiple instance Semi-Proximal Support Vector Machine) algorithm within a *Multiple Instance Learning* (MIL) framework. Each retinal image was represented as a *bag* of sub-region instances, with only a global label available per bag during training. Experiments on the public Messidor dataset provided encouraging results, but the simple RGB-based feature representation, capturing only mean and variance statistics of image sub-regions, left significant room for improvement.

Motivated by the success of **MultiExCam** [2], a hybrid explainable architecture recently developed by our group for skin lesion classification, we propose to adapt the same paradigm to DR detection, combining two well-established components into a novel pipeline. In the first stage, deep features from a pre-trained convolutional network are combined with hand-crafted color and texture descriptors to produce a richer image representation. In the second stage, these hybrid features feed the **mi-SPSVM** classifier within the MIL framework, whose core optimization problem remains unchanged, ensuring theoretical consistency and computational efficiency. Preliminary results will be discussed, with particular attention to the improvement over [1] in accuracy, sensitivity, and F-score.

References

- [1] Avolio M., Fuduli A., Vocaturo E., Zumpano E. (2023) On Detection of Diabetic Retinopathy via Multiple Instance Learning. In: *Proc. IDEAS*, pp. 170–176.
- [2] Ruga T., Caroprese L., Vocaturo E., Zumpano E. (2026) MultiExCam: A multi approach and explainable artificial intelligence architecture for skin lesion classification. *Computer Methods and Programs in Biomedicine*, vol. 273, 109081.

A Conformal Prediction Framework for Chance-Constrained Constraint Learning

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Keywords. Constraint learning; chance constraint; machine learning.

Optimization with Constraint Learning (OCL) embeds machine learning (ML) models as constraints within mathematical optimization problems when analytical formulations are unavailable. However, traditional OCL lacks formal probabilistic guarantees on constraint satisfaction: the optimizer may exploit prediction errors, producing solutions that appear feasible under the learned model but violate the true underlying constraint. Existing approaches to this problem rely on quantile regression to enforce chance constraints on learned variables, but their coverage guarantees are asymptotic and depend on correct model specification.

The proposed methodology employs conformal prediction to enforce chance constraints within the constraint learning framework. Conformal prediction provides distribution-free, finite-sample guarantees: given a calibration set and a confidence level $1 - \varepsilon$, it yields a margin γ such that the true constraint value lies within a γ -interval centered on the ML model prediction with probability at least $1 - \varepsilon$, regardless of the data distribution or model class. This margin is computed empirically from the prediction errors observed on a held-out calibration set, and is incorporated to tighten the learned constraint within the optimization, ensuring feasibility at the specified confidence level.

The framework is model-agnostic—compatible with any predictive model that admits mixed-integer reformulation—and can be combined with physics-informed learning strategies that embed known structural knowledge into the ML model to improve generalization. the proposed approach is validated on an energy-system scheduling problem involving learned constraints for both network-physics feasibility and equipment degradation.

Acknowledgements. This work contributes to the basic research activities of the WP9.6: “AI for Green” supported by the PNRR project FAIR - Future AI Research (PE00000013), Spoke 9 - Green-aware AI, under the NRRP MUR program funded by the NextGenerationEU.

Multi-modal Koiter-based ROM for Kirchhoff-Love Isogeometric Shells via Infinity Computer Differentiation

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Keywords. Buckling and post-buckling analysis; isogeometric analysis; infinity computer; numerical differentiation; numerical infinitesimals.

This work proposes an efficient Reduced Order Model (ROM) based on a numerical implementation of Koiter’s theory [1] of elastic stability. By utilizing a multi-modal asymptotic expansion to approximate displacement fields through a limited number of m buckling modes and their corresponding quadratic correctives, the equilibrium paths of both perfect and imperfect structures can be evaluated at a fraction of the standard computational cost.

The mechanical framework employs Kirchhoff-Love (KL) shell theory within Isogeometric Analysis (IGA). However, the complex kinematics and high degree of nonlinearity make the analytical derivation of energy variations—up to the fourth order—extremely difficult to handle.

To overcome these limitations, a novel numerical differentiation technique is integrated, leveraging the Infinity Computer (IC) framework [2]. This approach enables the computation of high-order derivatives by utilizing a specialized numbering system for infinite and infinitesimal quantities, that provides high-precision evaluation of derivatives without requiring explicit code generation. Numerical experiments demonstrate that this advanced differentiation tool offers a robust and accurate framework for the post-buckling analysis of shell structures.

References

- [1] Garcea G., Liguori F.S., Leonetti L., Magisano D., Madeo A. (2017) Accurate and efficient a posteriori account of geometrical imperfections in Koiter finite element analysis. *IJNME*, vol. 112, pp. 1154–1174.
- [2] Sergeyev Y. D. (2017) Numerical infinities and infinitesimals: Methodology, applications, and repercussions on two Hilbert problems. *EMS Surveys in Mathematical Sciences*, vol. 4, pp. 219–320.

Infinity Computer Differentiation for Multi-modal Koiter-based ROM

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Keywords. Infinity Computer; numerical differentiation; numerical infinitesimals.

This work proposes a new way to compute high order partial derivative using the Infinity Computer framework that generalize the algorithms for the computation of derivatives, Lie Derivatives, gradient vectors, Jacobian and Hessian matrices presented in [1, 2].

The Matlab class in [2] that uses a subset of the Infinity Computer Arithmetic has been integrated with routines for the computation of high order partial derivatives.

The application of the derived techniques to Reduced Order Model (ROM) based on a numerical implementation of Koiter's theory [3] of elastic stability show the potentiality of the derived framework.

References

- [1] Sergeyev Y.D. (2011) Higher order numerical differentiation on the Infinity Computer. *Optimization Letters*, vol. 5(4), pp. 575–585. DOI: 10.1007/s11590-010-0221-y.
- [2] Iavernaro F., Mazzia F., Mukhametzhanov M.S., Sergeyev Y.D. (2021) Computation of higher order Lie derivatives on the Infinity Computer *Journal of Computational and Applied Mathematics*, vol. 383, article 113135. DOI: 10.1016/j.cam.2020.113135
- [3] Garcea G., Liguori F.S., Leonetti L., Magisano D., Madeo A. (2017) Accurate and efficient a posteriori account of geometrical imperfections in Koiter finite element analysis. *IJNME*, vol. 112, pp. 1154–1174.

A Two-Point Stepsize Rule in Subgradient Method

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Keywords. Nonsmooth optimization; subgradient algorithm; stepsize rule; Barzilai-Borwein method.

We address convex nonsmooth optimization problems by introducing a novel stepsize strategy for the subgradient method. The proposed rule is inspired by the Barzilai-Borwein (BB) two-point stepsize, originally designed for smooth optimization. By integrating the BB logic with appropriate safeguarding techniques, we prove that the algorithm converges to a point satisfying an approximate optimality condition. To evaluate the efficiency of the method, we perform several numerical tests on classic nonsmooth benchmark functions and on large-scale dual problems arising from the Lagrangian relaxation of integer linear programs. The results show competitive performance compared to standard subgradient schemes.

References

- [1] Carrabs F., Gaudioso M., Miglionico G. (2024) A two-point heuristic to calculate the stepsize in subgradient method with application to a network design problem. *EURO Journal on Computational Optimization*, vol. 12, article 100092.

Method for Optimal and Drivable Train Trajectories

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Keywords. Optimal control; trajectory optimization; nonlinear programming; dynamic programming; energy efficiency; railways.

In railway operations, determining a trajectory of a train that ensures on-time arrival with minimal energy consumption is referred to as Energy Efficient Train Control (EETC). Existing EETC approaches typically consider only essential safety constraints, including nonlinear motion equations as equality constraints and limits on traction and braking forces as inequality constraints. Additionally, adherence to scheduled journey times is enforced through time bounds.

From a practical perspective, a trajectory is considered “drivable” if it satisfies further requirements such as smooth motion, realistic transitions between traction regimes, and vehicle-specific control actions. However, trajectories obtained through conventional EETC approaches, while energy-efficient, often fail to meet these criteria and are therefore unsuitable for real-world applications such as driver assistance or training systems.

This work presents the modeling and implementation of a dynamic programming (DP) approach for computing optimal train trajectories that are not only energy-efficient but also drivable, comfortable, and associated with reduced mechanical wear. Additional operational requirements are incorporated into the optimization problem through constraints and modifications of the objective function.

Different trajectory optimization strategies, including nonlinear programming and dynamic programming, are analyzed and compared. A direct trajectory optimization approach based on dynamic programming is selected and implemented. The resulting trajectories are evaluated against both real-world train data and solutions obtained from conventional EETC formulations.

The proposed framework extends the classical EETC problem toward a more comprehensive Optimal Train Control formulation that incorporates practical operational constraints, with potential benefits in terms of cost efficiency and applicability for railway operators.

Acknowledgements.

This work is part of a research collaboration between DB Regio and the Institute of Railway Engineering at the Technical University of Darmstadt.

Multiresolution Local Smoothness Detection in Non-Uniformly Sampled Multivariate Signals

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Keywords. Edge detection; scattered data; multiresolution analysis; local regularity.

Inspired by edge detection based on the decay behavior of wavelet coefficients, we introduce a linear-time algorithm for detecting the local regularity in non-uniformly sampled multivariate signals. Our approach quantifies regularity within the framework of microlocal spaces introduced by Jaffard. The central tool in our analysis is the fast samplelet transform, a distributional wavelet transform tailored to scattered data. We establish a connection between the decay of samplelet coefficients and the point-wise regularity of multivariate signals. As byproduct, we derive decay estimates for functions belonging to classical Hölder spaces and Sobolev-Slobodeckij spaces. While traditional wavelets are effective for regularity detection in structured and usually low dimensional data, they are not suited to process irregularly sampled or scattered data. Differently, samplelets demonstrate robust performance for scattered and even higher dimensional data. To illustrate our theoretical findings, we present extensive numerical studies detecting local regularity of one-, two- and three-dimensional signals, ranging from non-uniformly sampled time series over image segmentation to edge detection in point clouds.

This is a joint project with Sara Avesani and Michael Multerer [1].

Acknowledgements.

The authors have been supported by the SNSF starting grant “Multiresolution methods for unstructured data” (TMSGI2 211684).

References

- [1] Avesani S., Giacchi G., Multerer M. (2025) *Multiresolution local smoothness detection in non-uniformly sampled multivariate signals*. ArXiv preprint arXiv:2507.13480.

Approximation and Sensitivity Analysis of a Water Percolation Model

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Keywords. Percolation modeling; RBFs approximation; sensitivity analysis.

Water percolation in porous media is a process investigated from different viewpoints, given its relevance to many applied problems. Water percolation is essential in hydrogeology for studying water infiltrations with contaminant transport, landslide hazard triggered by rainfall, or replenishment of water tables. Moreover, water percolation is used in fields ranging from industry to engineering. Coffee extraction is an interesting case study with a strong socio-economic impact due to its widespread consumption and the global market it affects, as well as the potential benefits it offers both from a social and a health perspective.

The study of espresso coffee extraction focuses on the physical and chemical processes that arise mainly from the infiltration of high-temperature and high-pressure water into the ground coffee matrix. The resulting mathematical model [1] is a system of partial differential equations, mainly consisting of the Richard's equation (which includes the Darcy flux), advective-diffusive-reactive equations for the extraction or dissolution, transport, and diffusion of the main chemical species, and the convective-diffusive heat balance equation.

We propose a numerical approximation of the solution to the percolation model, through the collocation method with a spatial discretisation based on Radial Basis Functions (RBFs), i.e., the Kansa method. We also perform a sensitivity analysis of the model. The numerical solution obtained with the meshless technique will be shown and compared to that obtained using a well-known groundwater percolation software package. The preliminary results of the sensitivity analysis will also be presented, highlighting both the robustness of the proposed model with respect to some calibration parameters and its flexibility with respect to others.

References

- [1] Angeloni S., Giacomini J., Maponi P., Perticarini A., Vittori S., Cognigni L., Fioretti L. (2023) Computer percolation models for espresso coffee: State of the art, results and future perspectives. *Applied Sciences*, vol. 13(4), article 2688.

Poisoning Spherical Classifiers via Nonsmooth DC Optimization

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Keywords. Adversarial learning; poisoning attack; DC programming.

Adversarial attacks on machine learning models have been extensively studied in recent years [2], with poisoning representing a particularly critical threat since it corrupts the training data aiming to induce a model with degraded or manipulated behavior. We investigate a poisoning strategy that targets spherical classifiers, i.e., nonlinear models that identify a sphere in the feature space to separate two classes of instances. We assume that a trained spherical classifier has been obtained by minimizing a loss function on a labeled training set. Then, we study the problem of determining norm-constrained perturbations of a prescribed subset of training instances, so as to generate an alternative classifier, with the same radius, that is geometrically distant from the original one, while still correctly classifying the perturbed instances, up to a soft-margin error. We consider the displacement of the center as a natural measure of the attack’s impact and we formulate two complementary optimization problems. The first maximizes the displacement of the center subject to a budget constraint on the perturbation norms and classification requirements on the perturbed dataset. The second minimizes the total perturbation cost subject to a minimum displacement constraint ensuring maximum disagreement between the two classifiers. Both formulations involve Difference-of-Convex (DC) functions and naturally fit within the DC programming framework [1]. To solve such problems, we exploit bundle methods for nonsmooth DC optimization [3], which are well suited to handle the nonconvex and nonsmooth structure of the problem, and we provide preliminary numerical results on synthetic and real instances to illustrate the behavior of the proposed approach.

References

- [1] Astorino A., Gaudio M., Gorgone E., Manca B. (2025) Difference of Convex programming in adversarial SVM. *Journal of Computational and Applied Mathematics*, vol. 454.
- [2] Biggio B., Roli F. (2018) Wild patterns: Ten years after the rise of adversarial machine learning. *Pattern Recognition*, vol. 84, pp. 317–331.
- [3] D’Alessandro P., Gaudio M., Giallombardo G., Miglionico G. (2024) The descent-ascent algorithm for DC programming. *INFORMS Journal on Computing*, vol. 36(2), pp. 657–671.

Functional Autoencoder with Sobolev Spaces Embeddings

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Keywords. Operator learning; neural networks; autoencoders; Sobolev spaces.

Standard convolutional neural networks act as mappings between finite dimensional spaces with fixed dimensions where dimension size is often used as synonym to resolution. While for many imaging tasks it may be sufficient to work with fixed resolution, in physics-inspired problems one may want to vary the latter without necessity for re-training the models. In recent years there was a lot of progress in mesh-free modelling in machine learning, nevertheless it is still of interest to design generative architectures which can be tuned to the regularity of data (e.g., its Sobolev index of the appropriate space). In this work we propose a new resolution-invariant autoencoder architecture where encoder and decoder are composed of Neural Pseudo-Differential Operators. We discuss their respective Sobolev indices for standard neural architectures, discretization errors and also present numerical experiments on various 1D and 2D datasets (custom-made ones and PDE-inspired benchmarks).

References

- [1] Bunker J., Girolami M., Lambley H., Stuart A. M., Sullivan T. J. (2025) Autoencoders in function space. *Journal of Machine Learning Research*, vol. 26, no. 165:1–54.
- [2] Li Z., Kovachki N., Azizzadenesheli K., Liu B., Bhattacharya K., Stuart A., Anandkumar A. (2020) Neural operator: Graph kernel network for partial differential equations. *ArXiv preprint arXiv:2003.03485*.
- [3] Shin J. Y., Lee J. Y., Hwang H. J. (2022) Pseudo-differential neural operator: Generalized Fourier neural operator for learning solution operators of partial differential equations. *ArXiv preprint arXiv:2201.11967*.

Challenges in the Use of Iterative Schemes within Interior Point Methods

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Keywords. Optimization; interior point methods; preconditioners.

Due to the use of logarithmic barrier function, interior point methods (IPMs) have to solve very ill-conditioned KKT systems. When problems are too large for direct methods of linear algebra Krylov-subspace methods are usually applied. This raises several research questions:

- How the use of inexact Newton directions affects the complexity and practical performance of the method?
- When is it appropriate to terminate the iterative scheme?
- How to precondition the KKT system?

This talk will attempt to answer such questions. In particular, it will address some of the key difficulties in the design and use of preconditioners for KKT systems, and will mention several successful attempts made so far [1–4].

References

- [1] Cipolla S., Gondzio J., Zanetti F. (2024) A regularized interior point method for sparse optimal transport on graphs. *European Journal of Operational Research*, vol. 319, pp. 413–426.
- [2] Schork L., Gondzio J. (2020) Implementation of an interior point method with basis preconditioning. *Mathematical Programming Computation*, vol. 12, pp. 603–635.
- [3] Zanetti F., Gondzio J., (2023) An interior-point-inspired algorithm for linear programs arising in discrete optimal transport. *INFORMS Journal on Computing*, vol. 35, pp. 1061–1078.
- [4] Zanetti F., Gondzio J. (2023) A new stopping criterion for Krylov solvers applied in interior point methods. *SIAM Journal on Scientific Computing*, vol 45, pp. A703–A728.

A General Scaling Scheme for Lagrangian Relaxation in Structured Optimization Problems

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Keywords. Lagrangian relaxation; non-smooth optimization; logistics.

Lagrangian relaxation techniques are among the most effective methodologies for tackling large scale structured optimization problems, particularly when the sub-problems exhibits a decomposable form. We observed that the way relaxed constraints are scaled can substantially influence the behaviour of the associated Lagrangian dual, with significant effects on the numerical performance of the non smooth methods used to optimize it.

Motivated by this observation, we design a general scheme for scaling the Lagrangian relaxation. The method adjusts the weights assigned to the relaxed constraints through a systematic procedure that does not depend on any specific problem structure. As a result, it can be seamlessly integrated into a wide range of Lagrangian based decomposition approaches and applied across different problem classes. The goal is to improve the numerical behaviour of the non smooth algorithm, in particular by reducing the number of iterations and enhancing overall effectiveness.

The proposed approach was initially tested on the multicommodity network design problem [1], where it provided noticeable improvements in terms of lower bound quality and number of iterations compared with standard unscaled relaxations. Then, we tested additional families of structured optimization problems, and preliminary computational experiments are reported.

References

- [1] Frangioni A., Gendron B., Gorgone E. (2024) Separable Lagrangian Decomposition for Quasi-Separable Problems. In: Crainic T.G., Gendreau M., Frangioni A. (eds.) *Combinatorial Optimization and Applications*. International Series in Operations Research & Management Science, vol. 358. Springer, Cham, pp. 207–225. DOI: 10.1007/978-3-031-57603-4_10

One-Ego Unreliable Friends Model for Local Network Formation: Mean-Field Analysis

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Keywords. Ego network; mean field; computation social science; Dunbar theory.

Recently, the Unreliable Friends Model has been proposed to capture Dunbar circle formation in local social networks. In this work, we derive a master equation for a specific variant of this model (one-Ego Unreliable Friends Model), providing an analytical description via the density function of the tie weight distribution. We identify the stationary states of this equation and validate them against stochastic simulations. Numerical experiments reveal a remarkably high level of accuracy of our approach.

References

- [1] Dunbar R. I. M. (1993) Coevolution of neocortical size, group size and language in humans. *Behavioral and brain sciences*, vol. 16, pp. 681–694.
- [2] Dunbar R. I. M., Arnaboldi V., Conti M., Passarella A. (2015) The structure of online social networks mirrors those in the offline world. *Social networks*, vol. 43, pp. 39–47.
- [3] Sutcliffe A. et al. (2012) Social relationships and the emergence of social networks. *Journal of Artificial Societies and Social Simulation*, vol. 15, p. 3.

RBF-KAN with Data-Driven Shape Parameter Initialization for Scientific Computing

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Kolmogorov-Arnold Networks (KANs) [3] have emerged as a promising alternative to Multi-Layer Perceptrons (MLPs) for scientific computing, offering learnable univariate activation functions on network edges. For computational efficiency, recent variants replace B-splines with Radial Basis Functions (RBFs), yielding architectures such as FastKAN [2] and Free-RBF-KAN [1]. These RBF-KAN models have demonstrated competitive performance for both function approximation and physics-informed PDE solving.

The shape parameter governing the RBF kernel plays a crucial role in RBF-KAN performance, yet its selection remains largely unaddressed. Fixed parameters are suboptimal across problem types, while purely gradient-based adaptation, as in Free-RBF-KAN, is sensitive to initialization and prone to local minima. Drawing on surrogate cross-validation for meshfree RBF collocation [4], we propose a Leave-One-Out Cross-Validation (LOOCV) strategy for data-driven shape parameter initialization, grounding the selection procedure in classical RBF approximation theory without modifying the subsequent gradient-based training. We evaluate the approach across Gaussian, Matérn, and Wendland kernel families on benchmark functions, comparing against FastKAN, Free-RBF-KAN, and MLP baselines. LOOCV-guided initialization provides an improvement in terms of accuracy and convergence for various kernels tested. We further extend this framework to physics-informed RBF-KAN for PDE solving, showing performance of the method on standard PDE benchmarks.

References

- [1] Chiu S.-T., Cheung S.W., Braga-Neto U. et al. (2026) Free-RBF-KAN: Kolmogorov-Arnold networks with adaptive radial basis functions for efficient function learning. ArXiv preprint *arXiv:2601.07760*.
- [2] Li Z. (2024) Kolmogorov-Arnold networks are radial basis function networks. ArXiv preprint *arXiv:2405.06721*.
- [3] Liu Z., Wang Y., Vaidya S. et al. (2024) KAN: Kolmogorov-Arnold Networks. ArXiv preprint *arXiv:2404.19756*.
- [4] Marchetti F. (2024) A fast surrogate cross validation algorithm for meshfree RBF collocation approaches. *Applied Mathematics and Computation*, vol. 481, 128943.

Dealing with Large Numbers via Mathematical Infinite Procedures: A Comparison between Aristarchus of Samos and Archimedes of Syracuse

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Keywords. Distance from sun to moon; anthyphairetic; incommensurability.

This paper compares the scientific approaches of two ancient Greek scholars, Aristarchus of Samos (310-230 BC) and Archimedes of Syracuse (287-212 BC). Specifically, it seeks to demonstrate how these two works from ancient Greece operated primarily mathematically, focusing on physical and/or astronomical questions. This resulted in a hypothetical style that was unusual for the time, yet constituted a form of mathematical cosmology that derived specific numerical results from a limited set of assumptions. It is worth highlighting that this mathematized style was not the norm within the framework of Greek science, given that, for example, Hipparchus and Ptolemy used mathematics to serve the needs of astronomy, based on careful observation and geared toward practical as well as theoretical concerns. On the other hand, Aristarchus and Archimedes used astronomical problems as a domain in which to demonstrate the power of mathematics to address questions about the physical world.

In this paper, we will focus primarily on Aristarchus's work on the distance between the sun and the moon ([2]), in order to demonstrate the implementation of a mathematical style based on anthyphairetic procedures (for extracting the ratio between two magnitudes) and not only trigonometric calculations (as is often emphasized), that seek to reduce comparative calculations of large numbers. These procedures rely on strategies concerning the numerical incommensurability inherent in a conception of infinity that allows us to distinguish between what can be measured with precision and what cannot.

References

- [1] Archimedes (1956) *The Arenarius of Archimedes*. E.J. Dijksterhuis (Ed.). Brill, Leiden.
- [2] Aristarchus (1993) *On the Sizes and Distances of the Sun and the Moon*. In: Heath T. L. (Ed.) *Aristarchus of Samos: The Ancient Copernicus*. Reprint. Thoemmes Press, Bristol: England.

Birkhoff-Kellogg Type Results with Applications

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Keywords. Eigenvalue problem; affine cone; topological methods.

We present some classical and recent results of Birkhoff-Kellogg type. We illustrate their applicability in the context of ordinary, functional and partial differential equations subject to local, nonlocal and functional boundary conditions.

Acknowledgements.

This research was supported by the GNAMPA (INdAM) and by the Italian Ministry of University and Research's grant [DD 170 24.09.2025 - DM MUR 737_2021] supporting the University of Calabria for financing research in "Aree Disciplinari Sociali e Umanistiche", Department of Economics, Statistics and Finance [DD 248/2025 17.12.2025]. Project title: "Actuarial and decision-making models to support the planning and development of healthcare welfare in Southern Italy and Calabria".

References

- [1] Calamai A., Infante G. (2023) An affine Birkhoff–Kellogg type result in cones with applications to functional differential equations. *Mathematical Methods in the Applied Sciences*, vol. 46, pp. 11897–11905.
- [2] Calamai A., Infante G. (2025) On the solvability of parameter-dependent elliptic functional BVPs on annular-like domains. *Discrete Contin. Dyn. Syst. Ser. B*, Early Access, doi: 10.3934/dcdsb.2024193.
- [3] Infante G., Lucisano P. (2026) Eigenvalues of a third order BVP subject to functional BCs, *Topol. Methods Nonlinear Anal.*, to appear, [arXiv:2412.20985](#).
- [4] Infante G., Veltri G.A. (2026) An eigenvalue result for Hammerstein integral equations with sign changing nonlinearities and functional terms. *Discrete Dynamics: Theory and Applications*, to appear, [arXiv:2505.15382](#).

A New Case Study for Grossone-Based Model in Infinite Computing and Knuth's Vertical Arrows Arithmetic

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Keywords. Grossone; mathematics education; Teaching/learning methods and strategies.

This paper presents a pilot study, structured as a case study, based on previous educational research conducted by the authors. The study investigates the introduction of the computational arithmetic of infinity and Knuth's up-arrow notation to represent extremely large numbers. Although earlier studies involved older students with more advanced mathematical backgrounds, this work focuses on first-year Italian high school students. The choice of this audience is motivated by two considerations. First, infinity computational arithmetic, specifically the quantity which represents grossone is introduced as a symbol within literal calculus, temporarily abstracted from its broader epistemological implications. Second, Knuth's notation is presented in a simplified and modulated form that highlights its recursive structure, typically associated with more advanced mathematical contexts involving unimaginable numbers, which are "unimaginably large numbers." The paper analyzes the outcomes of both a zero-knowledge and a quasi-zero-knowledge instructional approach, discussing the methodologies adopted and the results obtained.

Acknowledgements.

This research was partially supported by Namirial S.p.A. and Dectarlab S.r.l.

References

- [1] Ingarozza F., Adamo M. T., Martino M., Piscitelli A. (2020) A Grossone-Based Numerical Model for Computations with Infinity: A Case Study in an Italian High School. In: Sergeyev Y. D., Kvasov D. E. (eds.) *Numerical Computations: Theory and Algorithms. NUMTA 2019*. Lecture Notes in Computer Science, vol. 1973 pp. 451–462. Springer, Cham.
- [2] Ingarozza F., d'Atri G., Iembo R. (2025) Unimaginable Numbers: a Case Study as a Starting Point for an Educational Experimentation. In: Sergeyev Y. D., Kvasov D. E., Astorino A. (eds.) *Numerical Computations: Theory and Algorithms. NUMTA 2023*. Lecture Notes in Computer Science, vol. 14478, pp. 113–126. Springer, Cham.
- [3] Sergeyev Y. D. (2017) Numerical infinities and infinitesimals: Methodology, applications, and repercussions on two Hilbert problems. *EMS Surveys in Mathematical Sciences*, vol. 4, pp. 219–320.

A Grossone-Based Model for Carboncettus Octagons Sequence Computing

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Keywords. Grossone; geometry; Fibonacci numbers.

This work concerns a famous family of octagons known as *Carboncettus octagons*. It is an infinite, discrete sequence of octagons constructed using a constructive approach that, in this specific case, involves Fibonacci numbers. The idea is to study the algebraic and geometric properties of this sequence of octagons using the formal support provided by the modeling and computation of infinity arithmetic, made possible by the quantity called grossone. Thanks to this formal support model and the well-known arithmetic properties of the Fibonacci sequence, we will see how it is possible to highlight the algebraic characteristics of geometric elements, such as the values of the areas and perimeters of the octagons themselves. In this way, an alternative solution for the study of infinite sequences of geometric figures will be presented. This solution is provided precisely by the model of formalization of infinity, known as grossone. The idea is therefore to highlight the relationships that geometric parameters have with the numbers of the Fibonacci sequence using a relational approach based on the computational model of the system grossone. Generally speaking, we will adopt an approach in which we can define a grossone-based model.

Acknowledgements.

This research was partially supported by Namirial S.p.A. and Dectarlab S.r.l.

References

- [1] Ingarozza F., Piscitelli A. (2025) Some Notes on a Continuous Class of Octagons. In: Sergeyev Y. D., Kvasov D. E., Astorino A. (eds.) *Numerical Computations: Theory and Algorithms NUMTA 2023*. Lecture Notes in Computer Science, vol. 14478, pp. 127–138. Springer, Cham.
- [2] Caldarola F., d’Atri G., Maiolo M., Pirillo G. (2020) The Sequence of Carboncettus Octagons. In: Sergeyev Y. D., Kvasov D. E. (eds.) *Numerical Computations: Theory and Algorithms NUMTA 2019*. Lecture Notes in Computer Science, vol. 11973, pp. 373–380. Springer, Cham.
- [3] Sergeyev Y. D. (2017) Numerical infinities and infinitesimals: Methodology, applications, and repercussions on two Hilbert problems. *EMS Surveys in Mathematical Sciences*, vol. 4, pp. 219–320.

From Apeiron to Grossone: Semantic Archaeology and the Substantiation of Infinity

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Keywords. Infinite; Semantics; Grossone.

This study explores how the semantic framing of infinity has historically dictated its understanding and measurability. The inquiry moves from classical models of privation: the Greek apeiron, perceived as formless void, and the Roman infinitus, understood as a practical extension not yet delimited. In both cases, infinity remained an attribute of lack—a boundary that eluded calculation. In the Middle Ages, this elusiveness became dogma: infinity was the exclusive perfection of God, unattainable by human reason. Yet, it is within this medieval context that a pragmatic solution for dominating large scales in the real world emerged: the Grosso. Originating as the denarius grossus ("thick penny"), this silver coin established itself through a mass and stability superior to the debased small denari. It was not an attempt to measure infinity, but rather to provide a solid form to higher orders of magnitude. The article highlights the compelling terminological coincidence with Y.D. Sergeyev's Grossone. Both terms draw from the same root of "consistency." With the Grossone, infinity ceases to be an absence of an end (the prefix in-) to become a solid number. By replacing "lack" with "mass," infinity finally acquires a linguistic and mathematical body, enabling an arithmetic that transforms a millenary limit into an object of determinate calculation.

Acknowledgements.

This research was partially supported by Namirial S.p.A. and Dectarlab S.r.l.

References

- [1] Caldarola F., Cortese D., d'Atri G., Maiolo M. (2020) Paradoxes of the infinite and ontological dilemmas between ancient philosophy and modern mathematical solutions. In Sergeyev Y.D., Kvasov D.E. (eds.), *Numerical Computations: Theory and Algorithms 3rd Int. Conf.*, Springer, LNCS Vol. 11973, pp. 358–372.
- [2] Maiolo M. R., Ivano M. (2025) Legal Systems and Fractals, Towards Infinity Computing. In Sergeyev, Y.D., Kvasov, D.E., Astorino, A. (eds.), *Numerical Computations: Theory and Algorithms 4th Int. Conf.*, Springer, LNCS Vol. 14478, pp. 316–323.
- [3] Sergeyev Y. D. (2017) Numerical infinities and infinitesimals: methodology, applications, and repercussions on two Hilbert problems. *EMS Surv. Math. Sci.*, Vol. 4, pp. 219–320.

Scenario-Based Variable-Density Numerical Modelling of Saltwater Intrusion Under Combined Climate Stressors

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Keywords. Variable-density flow; SEAWAT; coastal aquifer; climate change scenarios.

The increasing severity of saltwater intrusion (SWI) is mainly due to the impacts of climate change, particularly in coastal areas. Changes in hydraulic gradients influence the density-dependent relationship between freshwater and seawater, indicating the need for an integrated numerical modelling tool. Therefore, this study applies a three-dimensional SEAWAT model in the Groundwater Modelling System (GMS) to evaluate long-term salinity trends in a Mediterranean coastal aquifer subjected to projected climate-induced stressors. The study area extends approximately 9.0 km² with a vertical thickness of 30 m. Hydraulic head data from twenty-two wells and salinity data from twelve monitoring sites were used to calibrate the model. Eight transient scenarios of 100-year duration were developed using relative sea-level rise (0.2-0.6 m) including subsidence and flowrate reduction (10-30%). The results indicate a progressive inland migration of the saline interface with recharge reduction exerting a stronger influence than a moderate sea-level rise. Under worst-case combined scenarios, nonlinear salinity expansion can be expected. This modelling framework enables the quantitative assessment of climate-driven vulnerability of coastal aquifers through variable-density flow and transport simulations under different climate scenarios.

Machine Learning Based Optimal UAV Path Planning

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Keywords. UAV path planning; trajectory optimization; guided policy search; radar threat avoidance.

This study aims to build a learning-based safe path planning framework for a UAV (Unmanned Aerial Vehicle) operating in a radar-threatened environment. The objective is to learn a policy that can drive the UAV to a predefined goal area while respecting turn-rate limitations and reducing exposure to the radar zones. The methodology combines optimization-based expert trajectory generation with sequence-based policy learning to enable immediate actions in real-time scenarios.

Trajectory scenarios are generated and solved through an optimization model with corresponding constraints. This stage produces optimal guidance examples with a balance between path length and avoidance of radar threats. Subsequently, these solutions are converted into a machine learning dataset, which is used to train a recurrent neural network policy.

The most effective solution is a combination of expert data, filtered training paths, a GRU-based policy, fine-tuning focused on specific cases, and a capture-hold terminal correction strategy.

The results demonstrate that combining optimization with machine learning provides a reliable approach for safe UAV path planning in the presence of radar threats. The study highlights that the quality of expert trajectories and the behavior at the terminal state in the generated dataset significantly influence the performance of the learned closed-loop system.

References

- [1] Bertsekas D. P. (2017) *Dynamic Programming and Optimal Control*. Athena Scientific, Belmont.
- [2] Morales M. (2020) *Grokking Deep Reinforcement Learning*. Manning Publications, Shelter Island, NY.
- [3] Alpdemir M. N. (2022) Tactical UAV path optimization under radar threat using deep reinforcement learning. *Neural Computing and Applications*, vol. 34, pp. 5649–5664.
- [4] Zhang T., Kahn G., Levine S., Abbeel P. (2016) Learning deep control policies for autonomous aerial vehicles with MPC-guided policy search. In: *2016 IEEE International Conference on Robotics and Automation (ICRA)*, Stockholm, Sweden, pp. 528–535.

Numerical Analysis of a Second-Order State-Constrained Optimal Control Problem Resulting from an Economic Model with Inertia in Investment

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Keywords. Optimal control; state constraints; investment modelling.

This work is aimed at the study of optimal control within the area of mathematical modeling and production processes with investment inertia. The model of economic growth with inertia in investment and state constraints is analysed theoretically and numerically. The primary research tool is the indirect approach based on Pontryagin’s maximum principle.

Production processes are subject to constraints, which are affine, as is typical for economic models. The main constraint under consideration is a lower limit on fixed assets, or capital, and, due to the assumed investment inertia, is represented in mathematical terms as a second-order state constraint. Problems with such type constraints represent certain research challenges. First, the capital constraint is a significantly irregular constraint, allowing for singularity in the corresponding Lagrange multiplier. Second, taking into account the fact that the system is affine in the control variable, the control set in the form of a parallelepiped does not exclude singular control modes. The latter significantly complicates the search for extremals and solutions.

To overcome these theoretical and computational difficulties, a specific form of the maximum principle is employed. This form takes into account the higher-order flavour of the problem formulation [1]. In the article, we also compare the extremals computed within the indirect method with the numerical solution obtained by the direct method based on optimization package IPOPT [2].

Acknowledgements. The work was supported by the Russian Science Foundation, project no. 25-21-00306, <https://rscf.ru/project/25-21-00306/>.

References

- [1] Karamzin D., Pereira F. (2023) On Higher-Order State Constraints. *SIAM J. Control and Optim.*, vol. 61, pp. 1913–1933.
- [2] Wächter A., Biegler L. T. (2006) On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Math. Program.*, vol. 106(1), pp. 25–57.

On High-Precision Numerical Solutions of Ordinary Differential Equations Using Levi-Civita Numbers

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Keywords. Numerical solutions of ODE's; Legendre-Darboux integration techniques; computational applications of the Levi-Civita field.

Among the most classical numerical techniques used for solving ordinary differential equations (ODE's) are Runge Kutta (RK) methods which gained popularity because of their balance of accuracy and computational efficiency. Besides their easy implementation, these methods suffer from round-off errors; achieving high accuracy requires a large number of intermediate steps, which increases computational cost and reduces efficiency. Moreover, most classical optimized numerical methods like ode45 in MATLAB use floating-point arithmetic which limits accuracy because of finite precision, catastrophic cancelation, truncation errors, rounding errors and limits their use for long integration intervals. These limitations become severe in stiff ODE's, highly sensitive dynamic systems, or simulations that require extremely high precision.

To address these challenges, we investigate the use of Levi-Civita numbers. Recall that the Levi-Civita field $\mathcal{R}$ is the smallest non-Archimedean ordered field extension of the real numbers that is real closed and Cauchy-complete in the order topology. It is small enough so that its numbers can be used on a computer for computational applications; one such application is the fast and accurate computation of the derivatives of functions representable on a computer up to any order [2]. For an overview of $\mathcal{R}$, see [1] and the references therein.

We develop a computational framework for solving ODE's using the $\mathcal{R}$ arithmetic combined with Legendre-Darboux integration techniques. The Darboux formula combined with Legendre polynomials allows us to approximate an integral using our knowledge of the integrand and its derivatives at the end points of the interval of integration. We show with examples how our method surpasses the classical Runge-Kutta method in accuracy, stability, and robustness; and the attained accuracy can be made arbitrarily high.

References

- [1] Shamseddine K. (2015) Analysis on the Levi-Civita field and computational applications. *Journal of Applied Mathematics and Computation*, vol. 255, pp. 44–57.
- [2] Shamseddine K., Berz M. (1996) Exception handling in derivative computation with non-Archimedean calculus. In: *Computational Differentiation: Techniques, Applications, and Tools*, pp. 37–51. Philadelphia, 1996. SIAM.

Supply Chain Network Design Problem with Stochastic Vulnerability Constraints: Formulation and Branch-and-Cut Algorithm

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Keywords. Supply chains; reliable network design; generalized Erdős-Rényi random graphs; branch-and-cut.

The design and modelling of production-logistic systems and supply chains remains a significant challenge in modern operations research and management science. Generally speaking, the primary goal is to find a suitable balance between several opposing criteria including supply chain reliability and its overall costs. In this paper, we evolve the recent algorithmic network design approach subject to probabilistic vulnerability constraints proposed by Arslan and Laporte along with our own results for the Resilient Supply Chain Network Design Problem. We introduce the Supply Chain Network Design Problem with Stochastic Vulnerability Constraints (SCNDPSVC), which is aimed to construct a minimum cost sub-network tolerant to a wide range of potential disruptions modelled in terms of generalized Erdős-Rényi random graphs.

The objective of SCNDPSVC is to construct, for a given DAG H specifying the topology of a supply chain, digraph G establishing the production-logistic network, and probability threshold $\varepsilon > 0$, a minimum-cost subgraph G' of the graph G subject to: (i) graph H can be embedded into G' ; (ii) for any network disruption leading to the removal of an arbitrary set of arcs with joint probability at least ε , the surviving subgraph of G' still allows (backup) embedding of H under a mild relaxation of the resource constraints.

By extending the concept of problem-specific network cut, we propose for the SCNDPSVC a mixed-integer linear programming (MILP) formulation based on set-cover inspired master problem and original separation problems. In order to obtain (sub-) optimal solutions, we develop a branch-and-cut algorithm, whose computational efficiency is demonstrated through numerical evaluation.

Soil Salinity Mapping Using Multi-Source Remote Sensing Imagery and AI-Based Techniques

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Keywords. Soil salinity; remote sensing; machine learning.

Soil salinization is a critical natural hazard and a major environmental and agricultural challenge, particularly in arid and semi-arid regions, where elevated soil salt concentrations adversely affect soil structure, fertility, microbial activity, and crop productivity. Globally, salinity influences over 1.5 billion hectares of agricultural land, jeopardizing food security, rural economies, and sustainable land management. Traditional field-based assessment approaches, while accurate, are laborious, time-intensive, and spatially limited, impeding large-scale monitoring evaluations. To address these limitations, this study presents a framework that integrates ground measurements, remote sensing data, and artificial intelligence. A total of 150 soil samples, collected in March 2020, from the study area (Sibari plain, Calabria, southern Italy), were paired with high-resolution Sentinel-1 and multi-spectral imagery from the Sentinel-2 satellite. A comprehensive set of spectral indices was derived, and the most predictive were selected for soil salinity assessment employing recursive feature elimination (RFE). Subsequently, Machine Learning models, such as Random Forest (RF), Extreme Gradient Boosting (XGBoost), and Support Vector Machine (SVM), were utilized to predict Electrical Conductivity (EC). XGBoost model achieved the highest predictive accuracy ($R^2 = 0.82$), followed by RF ($R^2 = 0.73$) and SVM ($R^2 = 0.55$). Despite some inconsistencies when applied to unobserved seasons, the results highlight the strong potential of combining multi-source remote sensing and machine learning for operational soil salinity monitoring and high-resolution mapping of saline areas.

Acknowledgements.

This work was funded by the Next Generation EU - Italian NRRP, Mission 4, Component 2, Investment 1.5, call for the creation and strengthening of 'Innovation Ecosystems', building 'Territorial R&D Leaders' (Directorial Decree n. 2021/3277) - project Tech4You - Technologies for climate change adaptation and quality of life improvement, n. ECS0000009. This work reflects only the authors' views and opinions, neither the Ministry for University and Research nor the European Commission can be considered responsible for them.

Improvement of the Computation Algorithm of the Matrix Fractional Power Based on the Double-Exponential Formula

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Keywords. Matrix fractional power; numerical quadrature; convergence.

This study considers numerical calculation methods for the matrix fractional power. Matrix fractional power appears in some techniques of machine learning, such as weighted geometric mean [1] and neural learning algorithms [2].

For the computation of the matrix fractional power, methods based on integral representations have been gaining attention in recent years. The reason is that it is more suitable for sparse matrix calculations and parallel computing compared to iterative methods. Among such numerical integration-based approaches, Tatsuoka et al. [4] proposed a method based on the double-exponential formula [3]. They first applied a variable transformation $t = \phi(x) = e^{(\pi/2)\alpha \sinh x}$ to the integral representation of A^α ($0 < \alpha < 1$) as

$$A^\alpha = \frac{\sin(\alpha\pi)}{\alpha\pi} A \int_0^\infty (t^{1/\alpha} I + A)^{-1} dt = \frac{\sin(\alpha\pi)}{\alpha\pi} A \int_{-\infty}^\infty [\phi^{1/\alpha}(x) I + A]^{-1} \phi'(x) dx,$$

where the integral interval is infinite: $(-\infty, \infty)$. Then, they truncated the infinite interval to $[l, r]$ so that the error is less than or equal to the given tolerance ϵ . The truncated integral is approximated by the standard trapezoidal formula.

In this study, we improve their computation algorithm in the following two points: (i) a variable transformation and (ii) how to determine the target interval for computation and the step size. This improvement guarantees the theoretical convergence of the error and enables approximate calculations without computing the 2-norm of the matrix.

Acknowledgements.

This research was supported by the JSPS Grant-in-Aid for Scientific Research (C)(JP23K03218).

References

- [1] Fasi M., Iannazzo B. (2018) Computing the weighted geometric mean of two large-scale matrices and its inverse times a vector. *SIAM J. Matrix Anal. Appl.*, vol. 39, pp. 178–203.
- [2] Fiori S. (2008) Leap-frog-type learning algorithms over the Lie group of unitary matrices. *Neurocomputing*, vol. 71, pp. 2224–2244.
- [3] Takahasi H., Mori M. (1974) Double exponential formulas for numerical integration. *Publ. Res. Inst. Math. Sci*, Kyoto Univ., vol. 9, pp. 721–741.
- [4] Tatsuoka F., Sogabe T., Miyatake Y., Kemmochi T., Zhang S. L. (2021) Computing the matrix fractional power with the double exponential formula. *Electron. Trans. Numer. Anal.*, vol. 54, pp. 558–580.

Construction of the Variational Multiplier for Second-Order Partial Differential Equations with Deviating Arguments

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Keywords. Functional differential equations; inverse problem of the calculus of variations; conditions of potentiality, variational principles.

This work is devoted to solving the inverse problem of the calculus of variations for second-order partial differential-difference equations with a deviating argument. This work is a continuation of the study of the potentiality question for differential operators with deviating arguments. The main challenge for the class of operators under consideration is that, unlike potential operators, whose identification presents certain difficulties, non-potential operators have been little studied. It is precisely the lack of potentiality of an operator with respect to a given bilinear form that limits the application of classical variational principles. The paper proposes an algorithm for finding a variational multiplier, by which a previously non-potential operator becomes potential. Using the Helmholtz potentiality criterion for differential-difference operators, a theorem is proved.

A feature of the method is that the presence of an argument shift must be taken into account. The results obtained make it possible to reduce boundary value problems for such equations to the minimization of quadratic functionals, which allows the use of direct variational methods.

References

- [1] El'sgol'tz L.E. (1964) *Qualitative Methods in Mathematical Analysis*. Transl. Math. Monogr., vol. 12, Amer. Math. Soc.
- [2] Hale J. (1977) *Theory of Functional Differential Equations*. Springer Verlag, New York.
- [3] Tonti E. (1982) A general solution of the inverse problem of the calculus of variations. *Hadronic J.*, vol. 5(4), pp. 1404–1450.
- [4] Filippov V. M., Savchin V. M., Shorokhov S. G. (1994) Variational principles for nonpotential operators. *J. Math. Sci.*, vol 68(3), pp. 275–398.

CRLB-Based Configuration Analysis of Restricted Dirichlet-to-Neumann EIT Forward Model Statement

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Keywords. EIT; restricted DtN; CRLB; electrode configuration.

Electrical Impedance Tomography (EIT) reconstructs a conductivity from boundary measurements; the current-voltage protocol and electrode layout strongly affect the modelling result, motivating forward model configuration optimisation. We present a Cramér–Rao lower bound (CRLB) based sensitivity/configuration analysis for the *restricted* Dirichlet-to-Neumann (rDtN) EIT model of Klibanov [3], where data are available only on a subset of the boundary and the region of interest lies inside a surrounding background domain.

We derive the Fisher information matrix for the discretised rDtN parameters and use the resulting CRLB to adjust electrode configurations under the measurements. Jacobian-based sensitivity analysis is well established for full-boundary (DtN-type) EIT models [1, 2, 4]; our comparison shows that the same paradigm can be adapted to the restricted rDtN setting.

Numerical 2D FEM experiments demonstrate these effects on toy conductivity patterns with sharp inclusions and on labelled CT-derived conductivity maps, supporting rDtN compatible protocol design.

References

- [1] Adler A., Boyle A., Lionheart W.R.B. (2017) Efficient computations of the Jacobian matrix using different approaches are equivalent. In Proceedings of the *Conf. EIT 2017*, Dartmouth, NH (USA), p. 75.
- [2] Adler A., Boyle A. (2017) Electrical impedance tomography: tissue properties to image measures. *IEEE Transactions on Biomedical Engineering*, vol. 64(11), pp. 2494–2504. doi:10.1109/TBME.2017.2728323
- [3] Klibanov M.V., Li J., Zhang W. (2019) Convexification of electrical impedance tomography with restricted Dirichlet-to-Neumann map data. *Inverse Problems*, vol. 35(3), article 035005. doi:10.1088/1361-6420/aafecd
- [4] Gómez-Laberge C., Adler A. (2008) Direct EIT Jacobian calculations for conductivity change and electrode movement. *Physiological Measurement*, vol. 29(6), pp. S89. doi:10.1088/0967-3334/29/6/S08

Multi-Population Epidemic Control Problem Based on Deep Learning Approach

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Keywords. Epidemiology; mean-field control; generative adversarial network; physical informed neural network; optimization; inverse problem; regularization; COVID-19.

Multi-population mean-field control model (MP-MFC) provides a mathematically tractable framework for optimizing intervention strategies across heterogeneous populations in epidemiology [1, 2]. However, solving high-dimensional stochastic MP-MFC problems remains challenging due to complex cross-population interactions, curse of dimensionality, and stochastic epidemic dynamics. We propose a coupled adversarial network approach for numerical solutions to MP-MFCs in epidemiology. First, we establish a compartmental-structured MFC model capturing heterogeneous transmission dynamics (based on severe course of the disease), vaccination policies, and mobility-driven interactions between subpopulations [2, 3]. Leveraging the variational structure of MFCs, we reformulate the problem as a saddle-point optimization. Our architecture employs multiple generators to parameterize both population-specific controls and infection state distributions, while cooperative discriminators enforce consistency with epidemiological constraints and mean-field couplings based on PINNs. Adversarial training minimizes residuals of the coupled Hamilton-Jacobi-Bellman and Kolmogorov forward equations. We demonstrate the effectiveness of our approach for modeling of COVID-19 propagation in the Novosibirsk region.

Acknowledgements.

This research was supported by the grant of the state program of the “Sirius” Federal Territory “Scientific and technological development of the “Sirius” Federal Territory” (Agreement No. 26-03, date 07.07.2025).

References

- [1] Bensoussan A., Tao H., Lauriere M. (2018) Mean field control and mean field game models with several populations. *Minimax Theory Appl.*, vol. 3, pp. 173–209.
- [2] Wang G., Fang J., Jiang L., Yao W., Li N. (2024) Coupled alternating neural networks for solving multi-population high-dimensional mean-field games. *Mathematics*, vol. 12, 3803.
- [3] Petrakova V., Krivorotko O. (2025) Comparison of two mean field approaches to modeling an epidemic spread. *J. Optim. Theory Appl.*, vol. 204, 39.

Benchmarking Deterministic and Stochastic Methods for Constrained Global Optimization

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Keywords. Constrained global optimization; benchmarking tools; grossone.

This talk addresses methodological aspects of benchmarking deterministic and stochastic methods for constrained global optimization [1]. Particular attention is given to the challenges posed by nonlinear constraints and to the need for reliable and informative testing frameworks. Advanced *grossone*-based techniques for the development and testing of constrained global optimization methods are also discussed [2].

We review a range of benchmarking methodologies for global optimization algorithms, as, e. g., operational zones and aggregated operational zones. These tools provide insight into algorithmic behavior across different regions of the search space and varying levels of problem difficulty [3]. To support the development and systematic evaluation of constrained optimization methods, we consider a test problem generator proposed in [4]. This generator extends the classical GKLS framework—originally designed for box-constrained problems—to problems with general nonlinear constraints. It allows the controlled construction of test instances with tunable difficulty through the selection and combination of constraint types, enabling targeted assessment of algorithmic performance.

The proposed benchmarking tools are illustrated through numerical experiments involving several representative deterministic and stochastic methods for constrained global optimization dealing with low- and medium-dimensional problems and computationally expensive objective functions.

References

- [1] Strongin R. G., Sergeyev Y. D. (2000) *Global Optimization with Non-Convex Constraints: Sequential and Parallel Algorithms*. Kluwer Academic Publ., Dordrecht.
- [2] Sergeyev Y. D., De Leone R., eds. (2022) *Numerical Infinities and Infinitesimals in Optimization*. Springer, Cham.
- [3] Sergeyev Y. D., Kvasov D. E., Mukhametzhanov M. S. (2018) On the efficiency of nature-inspired metaheuristics in expensive global optimization with limited budget. *Scientific Reports*, vol. 8, article 453. DOI: 10.1038/s41598-017-18940-4.
- [4] Sergeyev Y. D., Kvasov D. E., Mukhametzhanov M. S. (2022) A generator of multiextremal test classes with known solutions for black-box constrained global optimization. *IEEE Transactions on Evolutionary Computation*, vol. 26(6), pp. 1261–1270. DOI: 10.1109/TEVC.2021.3139263.

Embracing the Uncertainty: Nondum, Three-Valued Logic, and Quantum Computation

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Keywords. Nondum; quantum computation; MV-algebras.

This talk introduces Nondum (η), a symbol for not-yet-known digits inside real numbers, and shows how it becomes a natural building block for a three-valued algebraic semantics of quantum computation. We emphasize η as a finite-information quantity that internalizes uncertainty directly into numeric syntax. We first recall the framework of MV-algebras, the algebraic semantics of Lukasiewicz logic, and the specific three-valued system MV3, defined over 0, 1/2, 1 with its characteristic strong and weak connectives. We then introduce MV3 η , obtained by replacing the static midpoint 1/2 with Nondum η , turning the intermediate truth value into an evolving propensity that can converge to 0 or 1 over time. This allows us to reinterpret strong conjunction and disjunction as algebraic mechanisms for informational cancellation and saturation. Using Mundici's correspondence between countable MV-algebras and approximately finite-dimensional C^* -algebras, we show how MV3 (and MV3 η) aligns with 2×2 complex matrices, i.e., the state space of a single qubit. The conclusion briefly sketches how this perspective could recast the structure of algorithms such as Grover's search and QAOA, and suggests directions for future work on MV3 η -based quantum models.

Acknowledgements.

This research was partially supported by Dectarlab S.r.l.

References

- [1] La Regina F., d'Atri G. (2025) Introducing Nondum, A Mathematical Notation for Computation with Approximations. In: Sergeyev, Y.D., Kvasov, D.E., Astorino, A. (eds.), *Numerical Computations: Theory and Algorithms 4th Int. Conf.*, Springer, LNCS Vol. 14478, pp. 301–308.
- [2] Caldarola F., Maiolo M. (2020) On the Topological Convergence of Multi-rule Sequences of Sets and Fractal Patterns. *Soft Computing*, vol. 24(23), pp. 17737–17749.
- [3] Sergeyev Y. D. (2017) Numerical infinities and infinitesimals: methodology, applications, and repercussions on two Hilbert problems. *EMS Surv. Math. Sci.*, Vol. 4, pp. 219–320.

Learning-Based Agent Logic for Robust Park-and-Ride Hub Location

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Keywords. Robust optimization; facility location; park & ride hubs; multi-agent system.

This research focuses on the development of agent logic within a Multi-Agent System (MAS) for solving the Park & Ride (P&R) hub location problem under uncertainty in customer behavior.

Each agent applies a reinforcement learning-based ranking mechanism to identify promising candidate locations for P&R hubs. Candidate locations are assigned adaptive rank values that guide probabilistic solution generation. Constructed solutions are evaluated using a problem-specific utility function, and location rankings are updated based on reward and advantage estimation.

Experimental results based on real-world data demonstrate that the proposed agent logic efficiently identifies promising candidate locations. When integrated into a multi-agent negotiation framework, these agents contribute to the identification of robust locations for P&R hubs.

To improve scalability and computational efficiency, the proposed MAS is deployed in a High-Performance Computing system, where agents are distributed across multiple computing nodes. This parallelization enables simultaneous solution generation and evaluation.

Acknowledgements.

This research has received funding from the Research Council of Lithuania (LMTLT), agreement No S-ITP-24-8, project title: Robust Optimization for Green Infrastructure Design using Artificial Intelligence (ROGIDAI).

Polynomial Approximation of Caputo Fractional Derivatives through the Constrained Mock-Chebyshev Least Squares Method

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Keywords. Caputo fractional derivative; polynomial approximation; constrained mock-Chebyshev least squares; Gauss-Jacobi quadrature.

Caputo fractional derivatives play a fundamental role in the mathematical modelling of fractional diffusion equations, since the prescribed initial data involve only integer-order derivatives, which are typically available in several applications, see e.g. [1,2]. In this work, we present a polynomial interpolation-regression procedure for the numerical approximation of Caputo fractional derivatives of order $\alpha > 0$ with $m - 1 < \alpha < m$, $m \in \mathbb{N}$, defined by

$$(D^\alpha f)(x) = \frac{1}{\Gamma(m - \alpha)} \int_0^x (x - t)^{m-\alpha-1} f^{(m)}(t) dt, \quad x \in (0, b].$$

The method is designed under the assumption that the target function f is known only through its evaluations on a grid of equispaced nodes. The approach combines interpolation at a suitably selected subset of equispaced nodes that mimic the Chebyshev-Lobatto distribution with a least squares regression process over the remaining data. The Caputo fractional derivative is then approximated by differentiating the reconstructed polynomial and by computing the associated weakly singular integral via Gauss-Jacobi quadrature formula, which guarantees exactness for the polynomial integrands involved. Numerical experiments demonstrate the accuracy of the proposed method.

References

- [1] Gao G., Sun Z., Zhang H. (2014) A new fractional numerical differentiation formula to approximate the Caputo fractional derivative and its applications. *J. Comput. Phys.*, vol. 259, pp. 33–50.
- [2] Jiang S., Zhang J., Zhang Q., Zhang Z. (2017) Fast evaluation of the Caputo fractional derivative and its applications to fractional diffusion equations. *Commun. Comput. Phys.*, vol. 21, pp. 650–678.
- [3] Dell’Accio F., Nudo F. (2024) Polynomial approximation of derivatives through a regression–interpolation method. *Appl. Math. Lett.*, vol. 152, p. 109010.

Product Integration Methods Based on Spline Quasi-Interpolation for Second-Kind Integral Equations with Weakly Singular Kernels

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Keywords. Splines; quasi-interpolation; weakly singular integral equations.

Spline quasi-interpolation is a well-established technique characterized by the accuracy of the approximation, the low computational cost and the straightforward implementation. Therefore, in the last years, this tool has been successfully employed also in the field of the approximation of the solution of second-kind integral equations with various type of kernels, provided that the exact solution of the considered integral problem has some continuous derivatives all over the domain (e.g. [1]). Regarding the integral equations with weakly singular kernels, it is well-known that, in general, the solution can be differentiable many times in the interior of the domain, while it can be only continuous at the boundary. It is clear that this inherent behavior negatively affects the convergence order of the classical numerical methods based on quasi-interpolation [2].

In this contribution we present two product integration schemes based on spline quasi-interpolation that preserve the high convergence order allowed by spline approximation also in presence of solutions not differentiable at the extrema of the domain. The first scheme relies on uniform spline quasi-interpolants applied to a regularized version of the integral problem, while the second one is constructed employing non-uniform quasi-interpolants over a proper graded mesh.

References

- [1] Aimi A., Leoni M.A., Remogna S. (2023) Numerical solution of Fredholm integral equations by spline quasi-interpolating projectors. In *International Conference on Mathematical and Computational Modelling, Approximation and Simulation* (ed by Sbibi D. et al.), SEMA SIMAI Springer series, Vol. 37, pp. 109–122.
- [2] Aimi A., Leoni M.A., Remogna S. (2024) Numerical solution of nonlinear Fredholm–Hammerstein integral equations with logarithmic kernel by spline quasi-interpolating projectors. *Mathematics and Computers in Simulation*, Vol. 223, pp. 183–194.

Hölder Condition and Global Optimization

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Keywords. Hölder and Lipschitz conditions; deterministic global optimization; space-filling curves; safe global optimization.

Global optimization plays an important role in a wide variety of applications, including electronics, engineering, machine learning, and optimal decision making. In this talk, global minimization problems of Lipschitz “black-box” functions over multi-dimensional domains are considered. One of the powerful ways to solve these problems proposes their reduction to equivalent one-dimensional problems satisfying Hölder condition using Peano-Hilbert space-filling curves (see [2, 4]).

To do this, it is necessary to choose three kinds of techniques: (i) ways of reduction of the multi-dimensional problem to the univariate case (namely, numerical approximations of the curves (see [2–4])); (ii) since very often the Hölder constant of the reduced one-dimensional problem is unknown, it is necessary to choose a way of its estimation; (iii) a global one-dimensional algorithm that will be used to solve the reduced problem.

Several algorithms choosing various techniques (i) – (iii) are proposed and discussed. In particular, Hölder global optimization algorithms inspired by Lipschitz geometric methods (see [1]) are introduced. Applications to robotics and safe global optimization are also briefly surveyed. Extensive numerical experiments on both benchmark test functions and real-world applications demonstrate the highly promising performance of the proposed methods.

References

- [1] Kvasov D. E., Sergeyev Y. D. (2012) Univariate geometric Lipschitz global optimization algorithms. *Numerical Algebra, Control and Optimization*, 2(1), 69–90.
- [2] Lera D., Sergeyev Y. D. (2018) GOSH: derivative-free global optimization using multi-dimensional space-filling curves. *Journal of Global Optimization*, 71(1), 193–211.
- [3] Sergeyev Y. D., Nasso M. C., Lera D. (2024) Numerical methods using two different approximations of space-filling curves for black-box global optimization. *Journal of Global Optimization*, 88(3), 707–722.
- [4] Sergeyev Y. D., Strongin R. G., Lera D. (2013) *Introduction to Global Optimization Exploiting Space-Filling Curves*. Springer, New York.

OpenACC Acceleration of Learning with Errors Key Encapsulation Mechanisms for Post-Quantum Cryptography

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Keywords. Post-quantum cryptography; key encapsulation mechanism; learning with errors; parallel computing.

In light of the threat posed by quantum computing [1], lattice-based cryptography, particularly Learning With Errors (LWE) [2] schemes, is a leading post-quantum candidate due to its apparent resistance to known quantum attacks. However, post-quantum schemes, such as [3], require more computational resources than the widely used RSA (Rivest-Shamir-Adleman) and Diffie-Hellman, creating a need for more efficient implementations. Unstructured LWE-based Key Encapsulation Mechanisms [3] (KEMs) are memory-bound and hard to parallelise: in particular, our OpenMP-based parallel implementation exhibits performance degradation at 32 threads for large lattice dimensions due to memory-bandwidth saturation and sequential random-number generation.

In this work, we propose a GPU-accelerated LWE-KEM using OpenACC. Our design executes encryption and decryption of all the bits of the message in batched kernels, enabling over one million concurrent threads and avoiding explicit data-transfer directives. Experiments show reduced memory allocation, data transfer, and random-number overhead compared with the CPU-based OpenMP version, achieving substantial speedups while preserving correctness and indicating that LWE schemes map efficiently to GPU parallelism.

References

- [1] Shor P.W. (1994) Algorithms for quantum computation: discrete logarithms and factoring. In: *Proceedings 35th Annual Symposium on Foundations of Computer Science*, Santa Fe (NM), USA, pp. 124–134.
- [2] Regev O. (2009) On lattices, learning with errors, random linear codes, and cryptography. *Journal of the ACM*, vol. 56(6), article 34.
- [3] National Institute of Standards and Technology (2023) *FIPS 203: Module-Lattice-Based Key-Encapsulation Mechanism Standard*. NIST.

Bayesian Calibration of Steel Creep Model with ACBICI

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Keywords. Bayesian calibration; creep modeling; uncertainty quantification.

Steel production accounts for approximately 20–25% of global industrial CO₂ emissions [1], making it one of the most carbon-intensive industrial sectors. However, model calibration remains challenging due to strong parameter coupling, nonlinear constitutive behavior, and experimental uncertainties, while conventional deterministic optimization methods often yield non-unique solutions and fail to quantify uncertainty.

To address these limitations, we develop a Bayesian calibration framework implemented in our open-source Python library, ACBICI (A Configurable Bayesian Calibration and Inference package) [2]. Inspired by the Bayesian inverse modeling principles of Kennedy and O’Hagan [3], the framework provides a flexible inference environment that can incorporate numerical models of varying complexity directly into the calibration process. Model parameters are sampled dynamically, and corresponding simulations are performed within an adaptive inference loop. Relevant response features are extracted from the simulation outputs and used to update posterior distributions, enabling robust parameter calibration and providing insight into the underlying model behavior.

The proposed framework shows promising predictive capability for steel creep behavior under uncertainty. By estimating posterior distributions of model parameters and hyperparameters that are consistent with experimental observations and numerical simulations, the methodology facilitates meaningful uncertainty quantification and may improve predictive reliability.

References

- [1] Clean Steel Partnership (2021) Strategic Research and Innovation Agenda (SRIA). ESTEP – European Steel Technology Platform, Brussels, Belgium. <https://www.estep.eu/assets/CleanSteelMembersection/CSP-SRIA-Oct2021-clean.pdf>
- [2] Schenk C., Romero I. (2026) A Framework for the Bayesian Calibration of Complex and Data-Scarce Models in Applied Sciences. ArXiv preprint *arXiv:2601.22890*. <https://arxiv.org/abs/2601.22890>
- [3] Kennedy M., O’Hagan A. (2001) Bayesian calibration of computer models. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, vol. 63, pp. 425–464. DOI: 10.1111/1467-9868.00294

Lens Methods as Geometric Probes Reveal Three Computational Phases in Large Language Models: The Rank Plateau Paradox

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Keywords. Mechanistic interpretability; lens methods; computational phases.

Transformer-based large language models (LLMs) compute through deep stacks of residual layers, widely interpreted as implementing *progressive representational refinement* [1]: each layer gradually distilling and compressing information toward the final output. We address this question geometrically via lens methods [2], linearized maps from intermediate hidden states to the output decoding space, studying their singular spectra and dominant subspaces across 11 models from three architectural families (Qwen, OLMo, and Gemma, 1B–32B parameters). Across all models and scales we find a consistent three-regime structure. An early alignment regime exhibits rapid subspace rotation and anomalous spectral norms. A much longer intermediate regime spans 70–90% of network depth, where the decoding subspace stabilizes (Grassmann similarity 0.87–0.95) while the effective rank rises, sustains a high plateau, and contracts only in the final convergence phase, which closes the computation with a sharp pivot toward the readout subspace. We term the intermediate behavior the **rank plateau paradox**: progressive refinement predicts simultaneous subspace convergence and rank compression, yet we observe a stable geometric frame maintaining a high-dimensional set of active directions, indicating additive enrichment rather than distillation. The three regimes emerge consistently across models, scales, and lens constructions. The intermediate regime width scales linearly with model depth and its barycenter remains fixed at 0.480 ± 0.012 independent of scale and architecture, suggesting that depth primarily extends a phase of additive enrichment within a stable geometric frame rather than accelerating representational refinement.

References

- [1] Shai A., Teixeira L., Oldenziel A., Marzen S., Riechers P. (2024) Transformers represent belief state geometry in their residual stream. *Advances in Neural Information Processing Systems*, vol. 37, pp. 75012–75034.
- [2] Belrose N., Furman Z., Smith L., Halawi D., Ostrovsky I., McKinney L., Biderman S., Steinhart J. (2023) Eliciting latent predictions from transformers with the tuned lens. ArXiv preprint [arXiv:2303.08112](https://arxiv.org/abs/2303.08112).

Sensitivity of High-Resolution Rain-on-Grid Simulations to Rainfall Spatial Variability: A Numerical Experiment on the Brague Catchment (France)

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Keywords. Flood events; rain-on-grid; GPU-accelerated modelling.

High-resolution numerical modelling is increasingly used to simulate flood dynamics generated by extreme hydrometeorological events [1]. In distributed hydrodynamic models, rainfall forcing remains a major source of uncertainty, as spatial and temporal variability in precipitation directly controls runoff generation and surface flow propagation [2]. With a focus on the Brague catchment (southeastern France) and the flood event of 23 November 2019, this study investigates the sensitivity of a fully distributed rain-on-grid hydrodynamic model to different representations of rainfall spatial variability. Flood patterns are simulated using the GPU-accelerated TUFLOW HPC solver, which applies an explicit finite-volume discretization of the two-dimensional shallow water equations on a structured Cartesian grid with 5 m resolution.

Six rainfall scenarios are tested, including lumped and distributed inputs from rain gauges and radar with varying resolutions. Results show limited differences between simulations, consistent with the low spatial variability of rainfall. These findings highlight the role of rainfall uncertainty and suggest that variability indicators can support more efficient flood modelling and nowcasting.

References

- [1] Costabile P., Costanzo C., Kalogiros J., Bellos V. (2023) Toward street-level nowcasting of flash floods impacts based on HPC hydrodynamic modeling at the watershed scale and high-resolution weather radar data. *Water Resources Research*, vol. 59(10), e2023WR034599.
- [2] Emmanuel I., Andrieu H., Leblois E., Janey N., Payrastre O. (2015) Influence of rainfall spatial variability on rainfall-runoff modelling: Benefit of a simulation approach? *Journal of Hydrology*, vol. 531, pp. 337–348.

Convergence Results for Stochastic Programming under Decision-Dependent Uncertainty

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Keywords. Stochastic optimization; decision-dependent uncertainty.

We study stochastic programs with decision-dependent uncertainty. That is

$$\inf_{x \in \mathcal{X}} \mathbb{E}_{\mathbf{P}(x)}[f(x, \xi)] \quad (1)$$

where $\mathcal{X} \subseteq \mathbb{R}^n$ is a set of feasible solutions and $f : \mathbb{R}^n \times \mathbb{R}^N \rightarrow \mathbb{R} \cup \{-\infty, \infty\}$ is a function of jointly x and a multivariate random variable ξ with values in $\mathbb{R}^N$. The distribution of the uncertain parameters ξ depends on the decision vector x through an arbitrary distribution map $\mathbf{P} : \mathcal{X} \rightarrow \mathcal{P}(\Omega, \mathcal{F})$ where $\mathcal{P}(\Omega, \mathcal{F})$ is a suitable collection of distributions on the measurable space $(\Omega, \mathcal{F})$. Problems of type (1) can, in some cases, be transformed into ordinary stochastic programming problems, see, e.g., [1], [2]. However, this entails loss of desirable properties of f . The vast majority of the instances of (1) are challenging and general-purpose solution methods are sparse.

We show that solutions to such problems, within arbitrary precision, can be obtained by means of (sequences of) piecewise-constant approximations constructed on partitions of the feasibility set $\mathcal{X}$. In particular, optimal objective values and solutions emerge in the limit as the resolution of the partition is refined. The concept is illustrated through numerical examples.

These results provide a new paradigm for solving arbitrary stochastic programs with decision-dependent uncertainty.

Acknowledgements.

This research was supported by the research grant NNF24OC0089770 from the Novo Nordisk Foundation.

References

- [1] Rubinstein R. Y., Shapiro A. (1993) *Discrete event systems: Sensitivity analysis and stochastic optimization by the score function method*. Wiley.
- [2] Pflug G. (1996) *Optimization of Stochastic Models: The interface between simulation and optimization*. Springer.

Extending Execution Transparency to Visualization in Parallel Cellular Automata Frameworks

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Keywords. HPC; transparent execution; cellular automata.

Cellular Automata (CA) are widely adopted to model complex spatial phenomena, ranging from geophysical hazards and hydrological processes to biological and crowd systems. As the scale and resolution of these simulations increase, CA execution increasingly relies on heterogeneous high-performance computing (HPC) environments, including shared-memory CPUs, distributed-memory systems, and GPUs. Although modern frameworks support transparent parallel execution across such architectures, visualization and post-processing remain a critical weak point: simulation outputs are often inspected through backend-specific tools or ad hoc scripts, making interactive analysis difficult to reproduce, port, and generalize across models. This paper presents a visualization integration layer for OpenCal++, aimed at extending execution transparency beyond computation and into the analysis stage. The proposed approach is based on a portable artifact-driven workflow that separates runtime execution from visualization through three components: an execution descriptor containing metadata about the simulation, per-timestep outputs storing the computed states, and a dynamically loadable model-specific binary exposing a stable interface for serialization and semantic interpretation of cell substates. This contract enables a generic Qt/VTK visualizer to reconstruct simulation domains, decode model-specific fields, and render them coherently without depending on backend-specific memory layouts or recompilation of the visualization application. The resulting system supports interactive timestep exploration, substate-aware inspection, configurable color mapping, and 2D/3D rendering strategies in which different fields can independently drive color and geometry. The approach is validated on representative CA applications, including crowd dynamics and debris-flow simulation, showing how the same visualization environment can support heterogeneous models while preserving semantic consistency. By making visualization portable, extensible, and independent of the execution context, the proposed framework represents a concrete step toward fully transparent HPC workflows for large-scale CA modeling.

Constructing Graphs from Neuroimaging Data: Methods for Static PET Data

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Keywords. Graph construction; brain networks; static PET imaging; similarity measures.

Graphs provide a natural mathematical framework for modelling the brain as a complex system. In large-scale brain networks, nodes correspond to brain regions of a predefined brain parcellation, while edges represent inter-regional relationships determined according to the type of data considered. In neuroimaging studies, graphs can be constructed from regional signals extracted from imaging data, each capturing distinct aspects of brain organization. Functional MRI provides regional blood-oxygen-level-dependent (BOLD) time series used to derive functional connectivity graphs reflecting temporal correlations between brain regions. Diffusion MRI measures water diffusion in brain tissue from which tractography reconstructs white matter tracts that define structural connectivity graphs representing anatomical pathways between regions. Positron emission tomography (PET) offers a complementary molecular imaging perspective on large-scale brain organization, enabling the in vivo quantification of regional substrates such as metabolism, neurotransmission, and proteinopathies. However, methodological standards for constructing single-subject graphs using static PET data remain limited. In this setting, the regional signal is represented by the distribution of voxel intensities within each brain region. We therefore investigate different strategies to define pairwise similarities between these distributions, including divergence measures such as Kullback-Leibler, Jensen-Shannon, and Hellinger, and to select the most informative edges, evaluating the resulting weighted undirected graphs through their predictive performance with respect to relevant clinical outcomes. Clarifying how graphs can be constructed from static PET data contributes to a more complete characterization of large-scale brain organization and opens the way to subject-specific multilayer graphs integrating complementary connectivity information across imaging modalities.

Spline-Based Reconstruction of Digital Breast Phantoms for *In Silico* Medical Imaging

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Keywords. Breast digital phantoms; *in silico* imaging; spline methods.

Anthropomorphic digital phantoms can be a valuable tool for *in silico* testing and optimization of screening and diagnostics techniques. These can be leveraged for virtual clinical trials and for the generation of synthetic medical images, via computational models simulating the entire image acquisition pipeline.

In this framework, we calculate realistic X-ray mammograms, using as an input 3D digital phantoms, reconstructed from sets of dedicated breast computed tomography (BCT) images from different patients [1]. After the application of *ad hoc* filters for reducing the level of noise in the source BCT images, we derive the phantom voxel-based structure and perform the segmentation into glandular, fat and skin tissues. To this aim, different approaches based on spline approximation or interpolation are compared, comprising the use of weighted essentially non-oscillatory (WENO) technique for handling sharp gradients and discontinuities [2].

To generate multiple phantoms from a single BCT scan, we also introduce geometrical transformations replicating breast shape deformities, and we locally add artificial lesions. After the simulation of phantom biomechanical compression, we model the X-ray interaction with breast tissues with a Monte Carlo approach to obtain synthetic mammograms with variable anatomical features.

Acknowledgements.

This research was supported by project 22HLT05 MAIBAI, financed by European Partnership on Metrology, and project CRT “Tecniche spline per la costruzione di modelli anatomici digitali nello screening mammografico”.

References

- [1] Oria M. et al. (2025) Image-based reconstruction of anthropomorphic breast phantoms for synthetic mammogram generation. *Comput. Biol. Med.*, vol. 198, 111121.
- [2] Arandiga F., Remogna S. (2026) Approximation of piecewise smooth functions by nonuniform non-linear quadratic and cubic spline quasi-interpolants. *Math. Comput. Simul.*, vol. 241, pp. 758–770.

Analyzing the Influence of Meteorological Factors on the Influx of Influenza-like Illness in Baguio City, Philippines

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Keywords. Cointegration analysis; influenza-like illness; outlier identification; outlier treatment.

Influenza-like illness (ILI), a syndrome that causes influenza and other respiratory diseases, can be attributed from the effects of meteorological factors in a certain area. Provided the climatic conditions in Baguio City, researchers conduct a cointegration analysis to study the interaction between weekly ILI cases and available meteorological factors from 2011 to 2023. Outliers present in ILI cases are identified through boxplot method, and are treated through an iterative process of winsorization and imputation. To reduce the possible number of outliers, data transformation methods are utilized in the study. As a result, temperature, humidity, and wind speed contribute to the frequency of ILI cases. To explore other possible factors, researchers recommend incorporating socioeconomic and geographic factors, and utilizing other multivariate time series methods on the given data.

Acknowledgements.

The researchers acknowledge the Baguio City Health Services Office - City Epidemiology and Surveillance Unit headed by Dr. Donnabel Tubera-Panes for the weekly ILI cases, and the Department of Science and Technology - Philippine Atmospheric Geophysical and Astronomical Services Administration for the weekly meteorological data.

References

- [1] Kelly H., Birch C. (2004) The causes and diagnosis of influenza-like illness. *Australian Family Physician*, vol. 33(5), pp. 305–309.
- [2] van Noort S. et al. (2012) The role of weather on the relation between influenza and influenza-like illness. *Journal of Theoretical Biology*, vol. 298, pp. 131–137.
- [3] Herzer D. (2019) The long-run effect of aid on health: evidence from panel cointegration analysis. *Applied Economics*, vol. 51(12), pp. 1319–1338.
- [4] He F. et al. (2017) Construction and evaluation of two computational models for predicting the incidence of influenza in Nagasaki Prefecture, Japan. *Scientific Reports*, vol. 7, article 7192.

Geometric Optics and Grossone, Towards an Algebraic Resolution of Singularities

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Keywords. Geometric optics; ABCD matrices; Grossone.

Classical geometric optics, while robust, frequently encounters mathematical boundaries when modeling systems at extreme physical scales. The standard Ray Transfer Matrix (ABCD matrix) formalism operates within the real field $\mathbb{R}$ and relies on the limit operator to describe scenarios such as sources at infinite distances or lenses with infinitesimal focal lengths. However, the limit process is inherently “destructive”: it collapses the relative growth rates of system parameters, often leading to numerical instability and the loss of matrix invertibility.

This presentation introduces an innovative alternative by leveraging Sergeyev’s Grossone-based numerical system (①). This methodology extends the scalar field to include specific infinite and infinitesimal numerals, enabling exact arithmetic operations on quantities typically treated as asymptotic tendencies. We demonstrate that by redefining optical distances and powers as Grossone-based numerals, the resulting ABCD matrices maintain their full algebraic integrity.

Acknowledgements.

This research was partially supported by Namirial S.p.A. and Dectarlab S.r.l.

References

- [1] Antoniotti L., Marotta C.M., Melicchio A., Papaleo M.A. (2025) The Cantor-Vitali function and infinity computing. In: Sergeyev, Y.D., Kvasov, D.E., Astorino, A. (eds.), *Numerical Computations: Theory and Algorithms 4th Int. Conf.*, Springer, LNCS Vol. 14478, pp. 232–239.
- [2] Marotta C.M., Melicchio A. (2025) Game theory presented to Italian high school students in connection with infinity computing. In: Sergeyev, Y.D., Kvasov, D.E., Astorino, A. (eds.), *Numerical Computations: Theory and Algorithms 4th Int. Conf.*, Springer, LNCS Vol. 14478, pp. 139–153.
- [3] Caldarola F., Maiolo M. (2020) On the Topological Convergence of Multi-rule Sequences of Sets and Fractal Patterns. *Soft Computing*, vol. 24(23), pp. 17737–17749.

Beyond Singularities: Non-Archimedean Models for Stable Coastal Morphologies

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Keywords. Logarithmic spiral; bay-headland models; Grossone.

The geometric characterization of headland-bay beaches in static equilibrium is a cornerstone of coastal engineering, traditionally dominated by the logarithmic spiral model. While effective for macroscopic planforms, this approach collapses at the infinitesimal scale of the diffraction point. At the headland tip, classical calculus encounters analytical singularities where curvature radii vanish and gradients diverge, triggering numerical instabilities and non-physical drift in long-term morphodynamic simulations.

This talk proposes an innovative computational framework to resolve these discontinuities by adopting a non-Archimedean numeral system based on the Infinity Computing paradigm. By leveraging multi-scale positional arithmetic, we move beyond the limitations of standard floating-point approximations. This model allows for the exact evaluation of spiral curvature at the origin and maintains a rigorous separation between macroscopic equilibrium and microscopic morphological updates. We demonstrate how this approach transforms the logarithmic spiral into a deterministic engineering tool, ensuring absolute numerical stability and a rigorous treatment of fractal coastal boundaries without resorting to empirical smoothing.

Acknowledgements.

This research was partially supported by Namirial S.p.A. and Dectarlab S.r.l.

References

- [1] Antoniotti L., Marotta C.M., Melicchio A., Papaleo M.A. (2025) The Cantor-Vitali function and infinity computing. In: Sergeyev, Y.D., Kvasov, D.E., Astorino, A. (eds.), *Numerical Computations: Theory and Algorithms 4th Int. Conf.*, Springer, LNCS Vol. 14478, pp. 232–239.
- [2] Caldarola F., Carini M., Maiolo M., Papaleo M. A. (2024) Iterative mathematical models based on curves and applications to coastal profiles. *Mediterranean Journal of Mathematics*, vol. 21, 178.
- [3] Sergeyev Y. D. (2017) Numerical infinities and infinitesimals: methodology, applications, and repercussions on two Hilbert problems. *EMS Surv. Math. Sci.*, vol. 4, pp. 219–320.

Preserving Clinical Entity Context in LLM-Generated Paraphrases of Clinical Notes

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Keywords. LLM; NLP; clinical text processing.

Large language models (LLMs) are increasingly being used to analyse complex textual data, including clinical documentation. However, establishing whether transformations generated by LLMs preserve the semantic and contextual meaning of the original text remains a significant challenge, particularly in the medical field, where even slight changes in text can impact clinical interpretation and subsequent information extraction. In this study, we examine the semantic accuracy of LLM-generated paraphrases of narrative clinical text from the MIMIC-III database [2]. Our proposed pipeline uses a locally deployed LLaMA model to rewrite clinical notes as if a senior physician were explaining them to a junior doctor [1]. The original and paraphrased notes are then automatically compared to produce a structured evaluation, which includes a **faithfulness score**, **missing information**, **hallucinated content**, and a **qualitative summary**. Preliminary results indicate that the framework captures the relationship between the original and the rewritten notes. For example, the statement “chest CT confirmed cavitary lesions in the left lung apex and pleural effusion” was paraphrased as “the patient underwent a chest CT revealing cavitary lesions in the left lung apex and pleural effusion”. The pipeline assigned a faithfulness score of 80 and produced a qualitative summary indicating preservation of the main clinical meaning, while also highlighting a plausible inferential addition, supported by the original context but not explicitly stated in the source text.

References

- [1] Inan H., Upasani K., Chi J., Rungta K., Iyer K., Mao Y., Tontchev M., Hu Q., Fuller B., Testuggine D. et al. (2023) Llama Guard: LLM-based input-output safeguard for human-AI conversations. *arXiv preprint arXiv:2312.06674*.
- [2] Johnson A. E., Pollard T. J., Shen L., Lehman L. W. H., Feng M., Ghassemi M. et al. (2016) MIMIC-III, a freely accessible critical care database. *Scientific Data*, vol. 3(1), pp. 1–9.

On the Potential Geophysical Fields Inversion

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Keywords. Forward and inverse problem; parallel algorithms.

This paper describes a method of potential geophysical fields interpretation, which is based on a stable inversion algorithm. Inverse problems of obtaining the distribution of physical parameters of the Earth's crust from a given field are ill-posed, which means that in general their solution is non-unique and unstable. Stability of the algorithms is ensured by the selection of an initial density model and by a step-by-step solution of the problem on the correctness set. We obtain a stable and meaningful solution of the inverse problem by calculating small additions to the initial model. Of practical interest are the problems of joint interpretation of geophysical fields, which satisfy similar equations and boundary conditions. Unified mathematical methods may be used for such cases. Potential fields, such as gravity, magnetic, and stationary heat-flow fields, satisfy the Poisson equation with a special type of inner sources and different boundary conditions. From the mathematical point of view, problems of calculating these fields are reduced to the numerical solution of potential boundary-value problems. Therefore, the theory and interpretation methods that were created for one of them work for the others as well.

The authors developed new effective methods for the complex interpretation of such data. These methods are based on the idea of localization of the effect that inhomogeneities in the medium occur on geophysical field values at the nearest points. We have developed software using parallel algorithms for the interpretation of the complex data. The iterative scheme, which changes the model slightly at each step, ensures that the geological meaning of the initial seismic data is preserved [1,2]. As a result, 3D models of the Earth's crust are obtained.

References

- [1] Martyshko P., Ladovskii I., Byzov D. (2021) Parallel Algorithms for Solving Inverse Gravimetry Problems: Application for Earth's Crust Density Models Creation. *Mathematics*, vol. 9, 2966. DOI: 10.3390/math9222966
- [2] Tsidaev A., Ladovskii I. (2024) On a Numerical Solution to an Inverse Structural Density Problem with a Method of Local Corrections. *Mathematics*, vol. 12(24), 3953. DOI: 10.3390/math12243953

Structured BPMN for AI Agent Orchestration

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Keywords. BPMN; AI Agents; Workflow Orchestration.

Business Process Model and Notation (BPMN) is widely used to model business processes and is traditionally interpreted through centralized token-based execution semantics [1,2]. In contrast, modern workflow orchestration platforms increasingly rely on networks of autonomous AI agents that communicate through explicit data dependencies and asynchronous message passing. This difference creates a conceptual mismatch between BPMN control-flow semantics and distributed agent-based execution.

This paper proposes a formal bridge between BPMN process models and AI agent dataflow orchestration. We formalize a structured subset of BPMN as a typed directed graph equipped with token-and-store operational semantics. To enable distributed execution, we introduce *Agentic Workflow Normal Form (AWNF)*, a normalization that enforces block-structured control flow, explicit task interfaces, guard locality, and a single-writer discipline for data objects.

Based on these constraints, we define *Agentic Dataflow Systems (ADS)*, where AI agents communicate through typed FIFO channels and execute tasks when the required control and data messages are available. A translation function Φ compiles BPMN models in AWNF into ADS networks by transforming control tokens into control messages and store-based dependencies into explicit data channels between agents.

We prove that the resulting ADS weakly simulates the BPMN operational semantics while preserving observable task execution traces. The proposed framework identifies the structural and data-discipline requirements that BPMN models must satisfy to support agent-based execution, providing a semantic foundation for integrating traditional process modeling with modern AI agent orchestration architectures [3,4].

References

- [1] Dumas M., La Rosa M., Mendling J., Reijers H. (2018) *Fundamentals of Business Process Management*. Springer.
- [2] Object Management Group, *Business Process Model and Notation (BPMN) Version 2.0.2*. OMG, 2014.
- [3] van der Aalst W. M. P. (2016) *Process Mining: Data Science in Action*. Springer.
- [4] Wooldridge M. (2009) *An Introduction to MultiAgent Systems*. Wiley.

Weighted Polynomial Approximation for Volterra-Fredholm Integral Equations on the Half-Line

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Keywords. Global approximation method; interpolation at Laguerre zeros; weighted spaces.

Volterra–Fredholm integral equations (VFIEs) arise in various physical, biological, and financial models, yet the case of unbounded domains has received comparatively less attention in the literature (cf. [1–3]).

We propose a global approximation method for VFIEs on the half-line based on weighted polynomial approximation and Lagrange interpolation at the zeros of Laguerre polynomials. More precisely, we consider equations of the form

$$f(x) - \left[\int_0^{+\infty} h(x, y) w(y) f(y) dy + \int_0^x k(x - y) w(y) f(y) dy \right] = g(x),$$

where $x \in (0, +\infty)$, f is the unknown function, $w(y) = y^\alpha e^{-y}$ with $\alpha > -1$, and k, h, g are given functions, with $k^{(i)}(0) = 0$ for $i = 0, \dots, r - 1$ and some $r \in \mathbb{N}$.

We prove stability and convergence in suitable weighted spaces and derive computable a priori error estimates in the weighted uniform norm, with rates depending on the regularity of the data.

Numerical experiments confirm the predicted convergence rates and show that the method is accurate and robust for this class of equations.

References

- [1] Brunner H. (2017) *Volterra integral equations*. Cambridge Monogr. Appl. Comput. Math., 30, Cambridge University Press, Cambridge.
- [2] Junghanns P., Mastroianni G., Notarangelo I. (2021) *Weighted Polynomial Approximation and Numerical Methods for Integral Equations*. Pathways in Mathematics, Birkhäuser/Springer, Cham.
- [3] Mastroianni G., Notarangelo I. (2026) A remark on a special class of Volterra-Fredholm integral equations *J. Comput. Appl. Math.*, 117119. <https://doi.org/10.1016/j.cam.2025.117119>

Permutations and Derangements: From Finite Towards Infinite Cases

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Keywords. Permutations; derangements; infinite computing.

This presentation explores the combinatorial properties of permutations and derangements, tracing their development from classical finite theory toward the new frontier of infinite computing. We begin with a historical and mathematical review of the subfactorial $!n$ and its fundamental recurrence relations, which characterize sets with a finite number of elements n .

The talk addresses the limitations of Cantor’s set theory, where the lack of arithmetical granularity assigns the same cardinality of the continuum to vastly different infinite structures, such as the symmetric group $S_{\mathbb{N}}$ and its various subsets. To overcome these ambiguities, we employ the grossone-based methodology, which allows for a precise, algebraic treatment of infinite quantities. By using this framework, we determine the exact and distinct numerosities for permutations and function spaces of infinite sets, including the set of natural numbers, the even numbers and more generally, the sets of multiples of some positive integer a .

The presentation concludes by providing preliminary highlights and computational hints on extending the concept of subfactorial to the infinite case.

Acknowledgements.

This research was partially supported by Namirial S.p.A. and Dectarlab S.r.l.

References

- [1] Antoniotti L., Marotta C.M., Melicchio A., Papaleo M.A. (2025) The Cantor-Vitali function and infinity computing. In: Sergeyev, Y.D., Kvasov, D.E., Astorino, A. (eds.), *Numerical Computations: Theory and Algorithms 4th Int. Conf.*, Springer, LNCS Vol. 14478, pp. 232–239.
- [2] Marotta C.M., Melicchio A. (2025) Game theory presented to Italian high school students in connection with infinity computing. In: Sergeyev, Y.D., Kvasov, D.E., Astorino, A. (eds.), *Numerical Computations: Theory and Algorithms 4th Int. Conf.*, Springer, LNCS Vol. 14478, pp. 139–153.
- [3] Caldarola F., Maiolo M. (2020) On the Topological Convergence of Multi-rule Sequences of Sets and Fractal Patterns. *Soft Computing*, vol. 24(23), pp. 17737–17749.

Globalizing DC Algorithms for Binary Linear Programming

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Keywords. Binary linear programming; DC programming; non-linear optimization.

We explore a continuous reformulation of Binary Linear Programming (BLP) by revisiting the classic concave programming approach. We apply an exact penalty function method to transform the discrete combinatorial problem into a continuous Difference of Convex (DC) optimization framework [1]. While DC programming offers a powerful set of tools for non-convex optimization, standard local algorithms, such as the Difference of Convex Algorithm (DCA), frequently become trapped in local minima, limiting their effectiveness. To address this, we develop and compare two distinct heuristics designed to escape these local optima, each grounded in a different theoretical foundation of global optimality. The first heuristic is based on the Hiriart-Urruty condition [2], characterizing the global minimum through the inclusion of ϵ -subdifferentials. When this condition is violated, the procedure identifies a global descent direction by sampling various directions and error tolerances (ϵ) to exit the current basin of attraction. The second heuristic, inspired by Strekalovsky's conditions [3], focuses on feasible boundary exploration. By projecting directions onto the constraints to find the furthest possible steps, this method identifies “jumps” to different parts of the feasible set. The efficacy of these two strategies is validated through numerical experiments on benchmark instances, providing a comparative analysis of their performance in terms of solution quality and global search capability.

References

- [1] Le Thi H. A., Pham Dinh T. (2005) The DC (difference of convex functions) programming and DCA revisited with DC models of real world nonconvex optimization problems. *Annals of Operations Research*, vol. 133, pp. 23–46.
- [2] Hiriart-Urruty J.-B. (1989) From convex optimization to nonconvex optimization. Necessary and sufficient conditions for global optimality. In: *Nonsmooth Optimization and Related Topics* 256, F.H. Clarke, V.F. Demyanov and F. Giannessi (eds.), Plenum Press, pp. 219–239.
- [3] Strekalovsky A. S (1997) On global optimality conditions for D.C. programming problems. Irkutsk State University Press, Irkutsk.

Sobol-Guided Bayesian Reduction for DPD in Evolving Systems

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Keywords. Digital predistortion; Bayesian optimization; Sobol indices; dimension reduction; evolving systems.

Digital predistortion (DPD) of nonlinear power amplifiers is a resource-constrained control problem in an evolving system: amplifier characteristics change with operating regime, temperature, and aging, so the predistorter must be repeatedly retuned [1, 2]. We consider discrete selection of DPD hyperparameters and minimize $J(\theta) = L(\theta) + \alpha S(\theta)$, where L is residual distortion and S penalizes multiplication count. Because each evaluation requires training and testing a DPD model, exhaustive search is expensive.

We combine Bayesian optimization with variance-based sensitivity analysis [3, 4]. A Gaussian-process surrogate with expected improvement explores the discrete architecture space, while total Sobol indices identify weakly influential coordinates. Under standard Sobol assumptions, if $\theta = (Z, \theta_i)$, the best L^2 predictor that ignores θ_i has irreducible error $\text{Var}(J)S_i^{(T)}$. Hence coordinates with small total effect can be removed with controlled loss. The resulting SOBRA algorithm freezes such coordinates by an L^2 -alignment rule and continues optimization in the reduced space.

Experiments on Chebyshev-based DPD design show that SOBRA reaches a near-best configuration in about half as many objective evaluations as standard Bayesian optimization and in tens of times fewer evaluations than grid search, while preserving comparable NMSE and favoring architectures with fewer multiplications. This supports Sobol-guided dimensionality reduction for resource-constrained optimization and control of evolving nonlinear systems.

References

- [1] Morgan D.R. et al. (2006) A generalized memory polynomial model for digital predistortion of RF power amplifiers. *IEEE Trans. Signal Process.*, vol. 54(10), pp. 3852–3860.
- [2] Kim J., Konstantinou K. (2001) Digital predistortion of wideband signals based on power amplifier model with memory. *Electron. Lett.*, vol. 37(23), pp. 1417–1418.
- [3] Snoek J., Larochelle H., Adams R.P. (2012) Practical Bayesian optimization of machine learning algorithms. *Adv. Neural Inf. Process. Syst.* 25, pp. 2951–2959.
- [4] Saltelli A. et al. (2010) Variance based sensitivity analysis of model output: design and estimator for the total sensitivity index. *Comput. Phys. Commun.*, vol. 181(2), pp. 259–270.

Repulsive Taxis Drive Spatial Patterns in Predator-Prey Models with Nonlocal Delay

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Keywords. Nonlocal delay; repulsive taxis; linear chain trick; pattern formation; Rosenzweig–MacArthur model.

In this study, we address a significant gap in current ecological theory by investigating how environmental memory modeled as a nonlocal time delay interacts with repulsive taxis to shape population dynamics. Unlike traditional models where pursuit-evasion strategies typically dampen oscillations [1], our work reveals a counterintuitive outcome: the avoidance behavior of predators navigating away from high prey density gradients can act as a powerful architect for spatial heterogeneity when mediated by delayed crowding effects. Using the linear chain trick [2], we reduce the complex integro-partial differential equation framework into a more tractable system of partial differential equations to perform a rigorous linear stability analysis. Our findings demonstrate that while nonlocal delay alone acts as a temporal buffer that stabilizes the uniform state, the introduction of repulsive taxis triggers a steady-state instability, leading to the formation of distinct spot patterns, a morphology previously undocumented in such taxis-delay systems. These results suggest that “risk-averse” foraging strategies, driven by factors like prey toxicity or collective defenses, provide a unique mechanism for creating stable ecological refuges.

Acknowledgements.

Dr. Mishra acknowledges the financial support received from the International Growth Centre, London School of Economics and Political Science, UK (project code: IND-25281). The author sincerely appreciates this support, which made the completion of this research possible.

References

- [1] Ainseba B., Bendahmane M., Noussair A. (2008) A reaction-diffusion system modeling predator-prey with prey-taxis. *Nonlinear Analysis Real World Applications*, vol. 9(5), pp. 2086–2105.
- [2] Hurtado P. J., Kiro Singh A. S. (2019) Generalizations of the linear chain trick: incorporating more flexible dwell time distributions into mean field ode models. *Journal of Mathematical Biology*, vol. 79(5), pp. 1831–1883.

Cardinality-Constrained Portfolio Optimization Methods

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Keywords. Portfolio optimization; cardinality constraint; mean-variance model; quadratic programming.

Portfolio optimization is a central problem in quantitative finance, commonly formulated using the mean-variance framework introduced by Harry Markowitz [1]. In practical investment settings, however, portfolios are often subject to additional constraints such as limits on the number of assets that can be included. This restriction, known as the cardinality constraint [2], transforms the classical portfolio optimization problem into a combinatorial and non-convex optimization problem. As a result, traditional quadratic programming [3] approaches become difficult to apply directly, particularly for large asset universes.

This paper reviews optimization methods for solving the cardinality-constrained portfolio selection problem. Both exact approaches, such as mixed-integer quadratic programming, and heuristic techniques including metaheuristic and approximation methods are discussed. The paper highlights the computational challenges associated with these models and analyzes the trade-off between solution quality and computational efficiency. Cardinality constraints enable the construction of more realistic and implementable portfolios by limiting excessive diversification and reducing transaction complexity.

Acknowledgements. This research was supported by the Czech Science Foundation Grant 26-22534S.

References

- [1] Markowitz H. M. (1999) The early history of portfolio theory: 1600-1960. *Financial Analysts Journal*, vol. 55, pp. 5–16.
- [2] Burdakov O. P., Kanzow C., Schwartz A. (2016) Mathematical programs with cardinality constraints: reformulation by complementarity-type conditions and a regularization method. *SIAM Journal on Optimization*, vol. 26, pp. 397–425.
- [3] Ketabchi S., Moosaei H., Hladik M. (2021) On the minimum-norm solution of convex quadratic programming. *RAIRO-Operations Research*, vol. 55, pp. 247–260.

Monte Carlo-Based Simulations of a Non-Markovian Open Quantum System: The Damped Harmonic Oscillator Case

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Keywords. Open quantum system; damped harmonic oscillator; non-Markovian noise; Monte Carlo; Euler-Maruyama.

The goal of this work is to present the numerical simulation of a non-Markovian open quantum system (a quantum sub-system open to interactions with an environment that introduces non-Markovian noise [1]) for which the analytical form of its time evolution and steady state solution are unknown in our case of interest. The specific non-Markovian open quantum system we will focus on is the case of the damped harmonic oscillator, which admits both a density matrix description [2] and a Wigner function formulation as well [3]. To solve numerically this non-Markovian open quantum system in the Wigner formulation, we will use a Monte Carlo based Euler-Maruyama method. The numerical simulation of its time evolution and the achieved numerical steady state will give guidance into the analytical solution to the Wigner steady state function. Our numerical results will be discussed and compared with the simulations of the observables discussed by Breuer et al. in [4].

References

- [1] Breuer H. P., Petruccione F. (2007) *The Theory of Open Quantum Systems*. Oxford University Press.
- [2] Lindblad G. (1976) On the generators of quantum dynamical semigroups. *Communications in Mathematical Physics*, 48(2) pp. 119–130.
- [3] Arnold A., Fagnola F., Neumann, L (2007) Quantum Fokker-Planck models: The Lindblad And Wigner approaches. In: *Quantum Probability and Related Topics* (ed. by J. C. García & R. Quezada), CIMAT (Mexico), pp. 23–48.
- [4] Breuer H. P., Kappler B., Petruccione F. (2001) The time-convolutionless projection operator technique in the quantum theory of dissipation and decoherence. *Annals of Physics*, vol. 291, pp. 36–70.

High-Speed Numerical Solution of Initial Value Problems for ODEs Using the IMT-DE Variable Transformation and FFT

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Keywords. Variable transformation; FFT; Sinc approximation.

This paper proposes a numerical solution method for initial value problems of ODE by applying the IMT-DE variable transformation [1] and the Fast Fourier Transform (FFT). The proposed numerical method can obtain approximate solutions faster than conventional methods [3], and its high accuracy is confirmed through multi-precision arithmetic.

This method considers an integral equation equivalent to the initial value problem of the ODE to be solved and solves it using Picard's iterative approximation. Solving the integral equation requires computing the integral operator, which is approximated using a numerical indefinite integral formula based on the IMT-DE variable transformation [2]. Furthermore, this numerical indefinite integration formula is based on a function approximation called the periodic Sinc approximation, which is essentially equivalent to the discrete Fourier transform. Therefore, the FFT can be applied to the numerical indefinite integration in order to reduce the computation cost composed with the direct evaluation. Numerical experiments using multi-precision arithmetic show that the proposed method achieves high accuracy while significantly reducing computational cost.

Acknowledgements.

This research was supported by JSPS KAKENHI Grant Number 24K06840.

References

- [1] Mori M. (1978) An IMT-type double exponential formula for numerical integration. *Publication of the Research Institute for Mathematical Sciences, Kyoto University*, vol. 14, pp. 713–729.
- [2] Ogata H. (2022) Numerical indefinite integration using the IMT-Type DE transformation and the Sinc approximation for periodic formations. *Transactims of the Japan Society for Industrial and Applied Mathematics*, vol. 32, pp. 1–19, DOI: 10.11540/jsiamt.32.1_1. (In Japanese.)
- [3] Ogata H. (2023) Numerical solution for initial value problems of ordinary differential equations by the IMT-DE type numerical indefinite integration formula. *Transactims of the Japan Society for Industrial and Applied Mathematics*, vol. 33, pp. 36–65, DOI: 10.11540.jsiamt.32.2_36. (In Japanese.)

A Metaheuristic Perspective on Dynamic Landscapes for Big Data Optimization

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Keywords. Metaheuristics; global optimization; dynamic landscapes; variable landscape search; formulation space; big data clustering.

Many large-scale optimization problems arising in big data environments involve extremely large search spaces and highly multimodal objective functions. Classical metaheuristics usually operate on a fixed objective function landscape and attempt to escape local optima through diversification strategies [1]. However, the static-landscape assumption may limit exploration in large-scale settings, including tasks such as information-context search in big text data.

This work discusses Variable Landscape Search (VLS), a metaheuristic paradigm in which the optimization landscape itself can change during the search process. Instead of optimizing a single objective function, VLS explores a family of related landscapes obtained through controlled variations of problem formulations or input data [2].

As a practical example, we interpret the Big-means clustering algorithm for Minimum Sum-of-Squares Clustering (MSSC) within the VLS framework. By repeatedly performing K-means on random data samples, Big-means effectively explores a sequence of correlated landscapes. This perspective explains its efficiency and illustrates how dynamic landscapes can support scalable optimization in big data environments [3].

Acknowledgements.

This research was supported by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (grant no. AP23486904).

References

- [1] Hansen P., Mladenovic N., Todosijevic R., Hanafi S. (2017) Variable Neighborhood Search: Basics and Variants. *EURO Journal on Computational Optimization*, vol. 5, 423–454.
- [2] Mussabayev R., Mussabayev R. (2025) Variable Landscape Search: A Novel Metaheuristic Paradigm for Unlocking Hidden Dimensions in Global Optimization. *Mathematics*, vol. 13, 3667.
- [3] Mussabayev R., Mladenovic N., Jarboui B., Mussabayev R. (2023) How to Use K-means for Big Data Clustering? *Pattern Recognition*, vol. 137, 109269.

University Students' Concept Images of Infinity

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Keywords. Infinity; Grossone methodology; concept images and concept definitions; mathematics education.

In this paper, we describe university students' concept images and definitions of mathematical infinity [1] in addition to their opinion of the Grossone methodology [2], which is a non-classical numerical method for dealing with infinity and infinitesimals proposed recently by the Italian mathematician Yaroslav Sergeyev.

The experimentation was carried out on university students specialized in pure or applied mathematics from 3 universities in Lebanon. It involved a questionnaire testing students' concept images and definitions of infinity, followed by an introductory session to the Grossone method, and later feedback on the new method introduced.

Results showed that even after being subject to advanced mathematical courses, conflicting cognitive factors related to infinity emerged. The Grossone method was positively received by university students who reported its ease and practicality in their feedback, despite some skeptical voices regarding its validity.

References

- [1] Tall D., Vinner S. (1981) Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, vol. 12, pp. 151–169.
- [2] Sergeyev Y. D. (2013) *Arithmetic of Infinity* (2nd ed.). Edizioni Orizzonti Meridionali, CS (1st ed.: 2003).

Fused Lasso for Spatial Reconstruction of Magnetic Particle Distribution using YIGM-Based Magnetorelaxometry

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Keywords. Yttrium-iron garnet magnetometer (YIGM); magnetorelaxometry (MRX); magnetic particles (MPs); Fused Lasso regularization.

Recently, solid-state yttrium-iron garnet film magnetometers (YIGMs) have emerged as a low-cost, fast recovery time, high-sensitivity alternative for the quantitative detection of magnetic nanoparticles in biomedicine. This work investigates the application of YIGMs in reconstructing the spatial distribution of magnetic particle (MP) concentrations within volumetric phantoms. Using magnetorelaxometry (MRX), a technique based on the relaxation behavior of magnetic particles after being magnetized by an external magnetic field. We address the MRX inverse problem via Fused Lasso regularization, which combines ℓ^1 -norm and Total Variation (TV) penalties, to achieve both sparsity and edge preservation in reconstructions, enhancing the accuracy and robustness. Experimental validation—performed with a YIGM and a coil excitation system—confirms that the fused Lasso accurately recovers MP distributions under varying external fields. These results highlight the potential of the YIG magnetometer as a powerful and accessible tool for magnetic imaging applications.

References

- [1] Koshev N. et al. (2024) Yttrium-iron garnet film magnetometer for registration of magnetic nano- and submicron particles: *In vitro* and *in vivo* studies. *IEEE Trans. Biomed. Eng.*, vol. 71(1), pp. 122–129.
- [2] Koshev N. et al. (2024) YIG-based sensor system for millisecond time range magnetorelaxometry. *IEEE Trans. Biomed. Eng.*, vol. 71(5), pp. 1640–1650.

Seasonal Forecasts over the Central Mediterranean Using Data-Driven Approaches

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Keywords. Seasonal forecasting; data-driven approaches; Mediterranean Basin.

Water scarcity and drought, intensified by climate change, are increasingly critical challenges to both economic stability and social well-being, particularly in vulnerable regions such as the Mediterranean Basin. Effective water resource management, therefore, depends on the availability of reliable and timely forecasting systems that enable the implementation of appropriate mitigation and adaptation strategies. In this context, seasonal forecasting has gained significant importance, especially with recent advances in Artificial Intelligence, which offer promising capabilities for improving the prediction and management of water scarcity and drought risks. In this study, a data-driven framework is employed to forecast precipitation 1- 6 months ahead using time series data collected from 134 local rain gauges distributed across the Calabrian Region in southern Italy. The methodology applied advanced machine learning techniques, including Gaussian Process Regression (GPR), Support Vector Machines (SVM), and Feed-Forward Neural Networks (FFNN). Model performance is systematically evaluated over the 2021-2023 period using statistical metrics, including bias, Root Mean Square Error (RMSE), and the Pearson correlation coefficient, across different spatial domains. The results indicate a strong concordance between model predictions and observed precipitation, with the data-driven approaches exhibiting particularly enhanced accuracy under wetter conditions.

Acknowledgements.

This study was funded by The Next Generation EU - Italian NRRP, Mission 4, Component 2, Investment 1.5, call for the creation and strengthening of 'Innovation Ecosystems', building 'Territorial R&D Leaders', Project Tech4You - Technologies for climate change adaptation and quality of life improvement, n. ECS0000009; The Next Generation EU - Italian NRRP, Mission 4, Component 2, Investment 1.3, project WaterWISE Water Management Strategies and Climate Change Adaptation in Southern Italy, n. PE00000005, CUP D43C22003030002.

Comparative Performance of Nonlinear and Traditional Input-Output Models: A Case Study of Kazakhstan

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Keywords. Input-output model; Young duality; optimization; resource allocation; economic equilibrium.

In the context of various shocks, developing new models for an endogenous analysis of risks in complex production networks becomes highly relevant. We discuss a novel approach based on a nonlinear modification [1] of the Leontief input-output model. This model enables factor substitution through neoclassical production functions, incorporates resource constraints, and determines economic equilibrium by considering inter-industry linkages, final consumption, and the allocation of primary inputs (imports, labor). It has the key advantage of being identifiable from publicly available input-output tables.

The model is formulated as a pair of mutually Young dual convex optimization problems. The primal problem allocates resources under constraints, while the dual relates price indices for goods to those for primary factors. Solving these yields the economic equilibrium under given scenarios. With CES functions, the equilibrium is computed explicitly, generating modified input-output tables that reflect factor substitution and network constraints. This enables scenario analysis and forecasting of inter-industry linkages under shocks.

We present an aggregated nonlinear input-output model for Kazakhstan calibrated with CES functions. Its performance is tested on official data (2012 - 2024) against a traditional fixed-cost structure model (Cobb-Douglas production). The nonlinear model yields lower error and captures macroeconomic dynamics more accurately under shocks, confirming its value for analyzing inter-industry linkages.

Acknowledgements.

Supported by the Ministry of Science and Higher Education, Kazakhstan (Science Committee, Grant No. AP32724353).

References

- [1] Obrosova N., Shananin A. (2024) A nonlinear Input-Output model with capacity constraints. *Siberian Electronic Mathematical Reports*, vol. 21(2), pp. 654–668.

Shifts in Competitive Equilibrium in Nonlinear Input-Output Models with Capacity and Financial Constraints

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Keywords. Competitive equilibrium; nonlinear input-output analysis; Young duality; optimization; resource allocation; equilibrium prices; CES function.

Nowadays, DCGE models based on Walrasian general equilibrium theory are a crucial tool for macroeconomic analysis and forecasting. However, their application faces challenges: empirical adequacy, verification difficulties, and scenario formulation due to many parameters with weak empirical justification. From the perspective of empirical adequacy, the input-output approach appears to be more reliable, as it relies on standardized input-output tables. Nevertheless, classical input-output models fail to account for agent behavior, assume linearity of inter-industry linkages, and do not incorporate factor substitution. This paper presents a framework for modified nonlinear input-output models with neoclassical production functions allowing input substitution. The model is formalized as an optimal resource allocation problem over a technology set defined by constraints. This approach partially addresses the challenge of representing agent behavior in input-output models. The technology set enables various resource constraints, such as capacity limits and financial restrictions. The primal convex optimization problem yields a dual problem of a special form, whose solution gives equilibrium prices in the production network. For CES production functions, the model allows explicit equilibrium calculation. Resource constraints are shown to shift equilibrium prices due to additional costs and markups arising from scarcity. This equilibrium shift mechanism opens prospects for using nonlinear input-output models as a tool for scenario analysis and forecasting the impacts of economic shocks and policy measures. Such models also offer a new approach for constructing and validating social accounting matrix (SAM) blocks, which provide the database for calibrating and specifying scenario conditions for DCGE models.

References

- [1] Obrosova N., Shanenin A. (2024) A nonlinear Input-Output model with capacity constraints. *Siberian Electronic Mathematical Reports*, vol. 21(2), pp. 654–668.

On the Role of Periodicity in IMT-type Variable Transformation and Periodic Sinc Approximation

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Keywords. Periodicity; variable transformation; Sinc approximation.

Variable transformations play a crucial role in high accuracy numerical computations such as the double exponential (DE) formula [3]. In this talk, we focus on the IMT-type variable transformation used in, for example, the IMT-type double exponential (DE) integration formula [1].

We claim that the periodicity that the IMT-type transformation generates makes numerical computations very accurate. In the IMT-type DE formula, we convert a desired integral to one over a finite interval, whose integrand is flattened at the endpoints, by the IMT-type transformation and evaluate numerically the transformed integral by the trapezoidal rule. The transformed integrand becomes a periodic C^∞ -class function, for which the trapezoidal rule is highly accurate due to the rapid decay of Fourier coefficients. Furthermore, the same periodic structure allows us to approximate transformed functions by the periodic Sinc approximation [2], whose error is small for smooth periodic functions. This viewpoint suggests broader applicability of IMT-type transformations to various numerical problems, including differential and integral equations.

Acknowledgements.

This research was supported by JSPS KAKENHI Grant Number 24K06840.

References

- [1] Mori M. (1978) An IMT-type double exponential formula for numerical integration. *Publications of the Research Institute for Mathematical Sciences, Kyoto University*, vol. 14, pp. 713–729.
- [2] Stenger F., Baker B., Brewer C., Hunter G., Kaputerko S., Shepherd J. (2008) Periodic approximation based on Sinc. *International Journal of Pure and Applied Mathematics*, vol. 49, pp. 63–72.
- [3] Takahasi H., Mori M. (1974) Double exponential formulas for numerical integration. *Publications of the Research Institute for Mathematical Sciences, Kyoto University*, vol. 9, pp. 721–741.

The Role of Generative Hyperparameters in the Philosophical Interpretation of Classical vs Relativistic Doppler in Redshift Calculations in Astrophysics

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Keywords. Artificial intelligence; philosophy of science; astrophysics.

We extracted the results of [4]. Through cross-correlation analysis, the authors obtained a radial velocity of 61,864 km/s with a Tonry & Davis R-value of 6.26. Considering the obtained radial velocity and applying the classical Doppler approximation, the measured redshift was $z = 0.206 \pm 0.001$, confirming an extragalactic nature of the source. Then we imagined what would happen if we have from the measurements the following parameters observed and calculated: the radial velocity and the Tonry & Davis R-value z redshift, and we would like to use an LLM studio to make a calculation using those values by tuning generative hyperparameters Temperature (#T) and Top-p in “Nucleus Sampling” [2]. Results: 1) with conservative #T and Top-p, the LLM performed a classical Doppler model and arrived at $z = 0.206$, the same as [4]. 2) Through “nucleus sampling”, the LLM performed a relativistic approximation, arriving at $z = 0.232$, also a correct value according to this paradigm. We argue that these hyperparameters function as epistemic regulators rather than mere technical settings, reshaping how scientific hypotheses are produced and constrained [3]. Also, laboratory instruments are not just tools to measure entities—they fundamentally shape our understanding of reality [1]—similarly, we can apply this idea to hyperparameters in LLMs. They affect the narratives we construct from the data. By tuning them, we engage in a process that influences how we interpret reality.

References

- [1] Hacking, I. (2000) *The Social Construction of What?* Harvard University Press, Cambridge (MA).
- [2] Hinton G., Vinyals O., Dean J. (2015) Distilling the Knowledge in a Neural Network. <http://arxiv.org/abs/1503.02531>
- [3] Longino H.E. (1990) *Science as Social Knowledge*. Princeton University Press, Princeton (NJ). <https://doi.org/10.1515/9780691209753>
- [4] Schmidt E. O. et al. (2025) High energy gamma-ray sources in the VVV survey – III. Spectroscopic confirmation of an AGN at low Galactic latitudes. *Monthly Notices of the Royal Astronomical Society*, vol. 544(1), pp. 152–159.

CME Tracking from LASCO Images: a Pipeline based on a Semi-Lagrangian Approximation

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Keywords. Coronal mass ejections; semi-lagrangian schemes; active contours.

Coronal mass ejections (CMEs) are considerable magnetized plasma eruptions from the corona of the Sun towards the heliosphere, often associated with solar flares. The analysis of these phenomena and the accurate tracking of the CMEs in real time is a fundamental and challenging issue since, if directed towards the Earth, they can cause geomagnetic storms and possibly disrupt radio transmissions and damage satellites and/or electrical transmission line facilities.

Since astronomical images are usually characterized by low resolution and significant presence of noise [1] due to the inherent limitations of the instrument sensors, and taking into account that the high-intensity foreground luminosity emanating from the Sun makes the visualization of the CME a non trivial task, in this work we propose a complete automatic pipeline to solve the problem, using a Semi-Lagrangian scheme [2] for the Chan-Vese model [3] to process the images and track the evolution curve. The input data are sequences of C2 and C3 images acquired by the Large Angle and Spectrometric Coronagraph (LASCO) (<https://www.swpc.noaa.gov/products/lasco-coronagraph>).

The implemented procedure is efficient and the results are reliable, in a fully automated real-time way. Comparisons with state-of-the-art methods are provided and confirm the good properties of the proposed pipeline.

Acknowledgements. This research was supported by the Italian Ministry of University and Research, under the PRIN2022 project entitled “Greedy Optimal Sampling for Solar Inverse Problems (GOSSIP)”, code 2022HFB32T_002, CUP J53C24002570001.

References

- [1] Roscani V., Tozza S., Castellano M., Merlin E., Ottaviani D., Falcone M., Fontana A. (2020) A comparative analysis of denoising algorithms for extragalactic imaging surveys. *A&A*, vol. 643, A43.
- [2] Falcone M., Ferretti R. (2014) *Semi-Lagrangian Approximation schemes for linear and Hamilton-Jacobi equations*, SIAM, Philadelphia, PA.
- [3] Chan T.F., Vese L.A. (2001) Active contours without edges. *IEEE Transactions on Image Processing*, vol. 10(2), pp. 266–277.

Exact k -Constrained Modularity Maximization with Branch and Bound

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Keywords. Global discrete optimization; modularity maximization; branch and bound.

Graph clustering is a task that aims to partition the vertices of a graph in such a way that closely connected vertices end up in the same cluster, and sparsely connected vertices end up in different clusters. Modularity maximization is one of the most popular techniques used for graph clustering. While the general problem is unconstrained, we focus on the k -constrained modularity maximization problem with the idea that some may be interested in finding optimal solutions with a certain number of clusters or that this constraint may help to find a solution faster. However, even a bounded problem is NP-hard, which makes it difficult to solve for large graphs.

We present an exact k -constrained modularity maximization parallelized branch and bound algorithm based on depth first search. We use various techniques to make the algorithm efficient, such as symmetry breaking, vertex preordering, and incrementally updating the objective function and upper bound. To calculate the initial lower bound, we used a greedy algorithm multiple times, which finds a close to optimum estimate. We also present a model on which we created a *gurobi* solution to compare against, because the *gurobi* solver is leading in the MIQP class of problems. We conducted various experiments on all algorithms on LFR and ABCD benchmark graphs, as well as some additional graphs, and compared the results.

Emergy Based Analysis for Sustainable Coastal Management: A Data Driven Decision Support Approach

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Keywords. Emergy analysis; Nature-Based Solutions (NBS); coastal zone management.

Coastal zones are increasingly threatened by the convergence of climate change, anthropogenic pressure, and erosion. Traditional economic assessments for shoreline interventions often fail by focusing solely on marketable manufactured items, ignoring the critical environmental work performed by the Geobiosphere. This talk presents a balanced sustainability decision-making framework based on *Emergy Analysis*, applied to a representative highly-stressed Mediterranean site in Calabria, Italy.

The research evaluates three distinct coastal protection strategies: interlocking concrete-block revetments, submerged natural rock breakwaters, and Nature-Based Solutions (NBS) comprising artificial reefs and *Posidonia Oceanica* meadows. By quantifying the environmental energy embodied in natural processes and materials, the study reveals that while hard engineering structures generally exhibit low sustainability and renewability indexes, NBS provide significantly higher ecological performance. In contrast, alternatives involving high material transformation and intensive construction render the lowest sustainability values. This work demonstrates that transitioning toward “softer” integrated designs is essential for the sustainable management of coastal materials and services, offering a robust thermodynamic alternative to classical economic approaches in maritime infrastructure planning.

References

- [1] Maiolo M., Mel R. A., Sinopoli S. (2020) A Stepwise Approach to Beach Restoration at Calabaia Beach. *Water*, vol. 12(10), 2677.
- [2] Maiolo M., Mel R. A., Sinopoli S. (2020) A Simplified Method for an Evaluation of the Effect of Submerged Breakwaters on Wave Damping: The Case Study of Calabaia Beach. *Journal of Marine Science and Engineering*, vol. 8(7), 510.

Extraction of Functional Brain Networks from EEGs: Classification of PNES and Epilepsy Patients

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Keywords. EEG; network analysis; machine learning.

Psychogenic non-epileptic seizures (PNES) pose a significant diagnostic challenge due to their clinical similarity to epileptic seizures (EPI) and the absence of clear epileptiform activity in standard EEG recordings [1]. Recent studies suggest that EEG functional connectivity and graph-based representations of brain activity may provide useful biomarkers for characterising alterations in the organisation of brain networks [2]. In this work, phase-based connectivity matrices, including Phase Lag Index (PLI), Phase Locking Value (PLV), and Weighted Phase Lag Index (WPLI), are computed from resting-state EEG signals and used to construct functional brain networks using different graph construction strategies, namely Minimum Connected Component (MCC), Minimum Spanning Tree (MST), and Orthogonal Minimum Spanning Tree (OMST). The organisation of these networks is characterised by extracting graph metrics, including global and local efficiency, clustering coefficient, path length, and betweenness centrality. These features are then used as input to supervised machine learning models to address a multi-class classification problem involving healthy controls (CNT), patients with PNES, and epileptic patients. In particular, Random Forest (RF) and Support Vector Machine (SVM) classifiers are employed to perform the classification task and evaluate the discriminative potential of the network-derived features.

Acknowledgements. V. Popello and I. Lazzaro contributed equally to this work.

References

- [1] Zucco C., Calabrese B., Mancuso R., Sturniolo M., Gambardella A., Cannataro M. (2022) Resting-State EEG Classification for PNES Diagnosis. In: *Proceedings of the International Conference on Computational Science (ICCS)*, Springer, pp. 526–538.
- [2] Roshanaei M., Norouzi H., Onton J., Makeig S., Mohammadi A. (2025) EEG-based functional and effective connectivity patterns during emotional episodes using graph theoretical analysis. *Scientific Reports*, 15(1), 2174.

SOE-Accelerated Alikhanov-Type Schemes and Physics-Informed Neural Network-Based Algorithms for Multi-Term and Distributed-Order Time-Fractional Reaction-Diffusion Equations

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Keywords. Multi-term Caputo derivative; fitted Alikhanov scheme; physics-informed neural network.

We develop fitted mesh-based and sum-of-exponentials (SOE) accelerated Alikhanov compact alternating direction implicit (ADI) schemes for multi-term and distributed-order two-dimensional time-fractional reaction-diffusion equations with weak initial singularities. Such fractional models play an important role in describing nonlocal transport, anomalous diffusion, viscoelastic behavior, and other memory-driven processes in physics, engineering, and biological systems [1].

A key contribution is a high-order Alikhanov-type approximation for the Caputo derivative at a variable super-convergent point, extending the idea of [2] to general nonuniform meshes. Combined with a fitted temporal mesh, this yields sharp local truncation estimates for low-regularity solutions and achieves a temporal convergence of order higher than in the existing works. Based on this approximation, we construct compact ADI schemes and further accelerate them via the SOE technique to significantly reduce memory usage and computational cost. Stability and convergence are rigorously established, and numerical experiments validate the theory and demonstrate the efficiency of the developed methods. We also utilize the proposed accelerated Alikhanov-type approximations to develop novel physics informed neural network based algorithms for challenging multi-term time-fractional problems, yielding a unified framework that links high-order numerical discretization with scientific machine learning. Numerical results further demonstrate the effectiveness of the proposed algorithms.

References

- [1] Podlubny I. (1999) *Fractional Differential Equations*. Mathematics in Science and Engineering, vol. 198. Academic Press, San Diego.
- [2] Gao G.-H., Alikhanov A. A., Sun Z.-Z. (2017) The temporal second order difference schemes based on the interpolation approximation for solving the time multi-term and distributed-order fractional sub-diffusion equations. *Journal of Scientific Computing*, vol. 73(1), pp. 93–121.

Bespoke Multiresolution Analysis of Graph Signals

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Keywords. Graph signal analysis; multiresolution analysis; compressed sensing; Isomap; samplelets.

We present a novel framework for discrete multiresolution analysis of graph signals. The main analytical tool is the samplelet transform, originally defined in the Euclidean framework as a discrete wavelet-like construction, tailored to the analysis of scattered data. The first contribution of this work is defining samplelets on graphs. To this end, we subdivide the graph into a fixed number of patches, embed each patch into a Euclidean space, where we construct samplelets, and subsequently pull the construction back to the graph. This ensures orthogonality, locality and vanishing moments with respect to properly defined polynomial spaces on graphs. Compared to classical Haar wavelets, this framework broadens the class of graph signals that can efficiently be compressed and analyzed. Along this line, we provide a definition of a class of signals that can be compressed using our construction. We support our findings with different examples of signals defined on graphs whose vertices lie on smooth manifolds. For efficient numerical implementation, we combine heavy edge clustering, to partition the graph into meaningful patches, with landmark **Isomap**, which provides low-dimensional embeddings for each patch. Our results demonstrate the method’s robustness, scalability and ability to yield sparse representations with controllable approximation error, significantly outperforming traditional Haar wavelet approaches in terms of compression efficiency and multiresolution fidelity.

Acknowledgements.

The authors have been funded by the Swiss National Science Foundation starting grant “Multiresolution methods for unstructured data” (TMSGI2_211684).

Uniform Asymptotic Approximation and Numerical Evaluation of the Reverse Generalized Bessel Polynomial Zeros

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Keywords. Asymptotic expansions; turning point theory; WKB theory; Bessel polynomials; numerical algorithms.

The zeros of Bessel polynomials have numerous applications in applied mathematics and engineering. They are used, for example, in the field of electronic engineering for the design of signal processing filters.

In this talk, we present uniform asymptotic approximations for the real and complex zeros of the reverse generalized Bessel polynomials $\theta_n(z; a)$ for large degree n and real parameter a . The expansions are uniformly valid for all the zeros and are highly accurate for moderate or large values of n .

The accuracy of the approximations for the complex zeros is tested using a method (implemented in Maple) for finding the complex zeros of solutions of second-order ODEs [1]. A fixed-precision implementation of this method for reverse generalized Bessel polynomials will also be discussed in the talk.

The numerical algorithm (highly efficient) incorporates the asymptotic approximations as starting values to later compute the remaining complex zeros of the function using Taylor series [2].

References

- [1] Segura J. (2013) Computing the complex zeros of special functions. *Numer. Math.*, vol 124(4), pp. 723–752.
- [2] Dunster T. M., Gil A., Ruiz-Antolin D., Segura J. (2026) Uniform asymptotic approximation and numerical evaluation of the reverse generalized Bessel polynomial zeros. *ETNA*. (In press.)

Global Approximation Methods for Terminal Value Problems of a Nonlinear Fractional Differential Equation

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Keywords. Fractional differential equation; Hammerstein equation; polynomial approximation.

Differential equations of fractional order are a topic of great current interest. They are useful tools for modeling various phenomena in science and engineering. These problems typically have the form

$$D_a^\alpha y(t) = f(t, y(t)), \quad y(a) = y_0 \quad (1)$$

where α is positive real number, D_a^α is the Caputo fractional differential operator, $t \in [a, b]$, f is known nonlinear function of t . However, from an application perspective, these methods are sometimes of limited use because they depend on the exact state of the process at the initial time $t = a$, and this may be impossible to determine in practice. If the value of $y(b)$ can be known only for some $b > a$ but not at a itself, this leads to the problem

$$D_a^\alpha y(t) = f(t, y(t)), \quad y(b) = y^* \quad (2)$$

and thus the problem becomes to solve the equation in $[a, c]$, with $c \geq b$ [1]. It is well known that fractional differential equations can be reformulated in integral forms. In the case of equation (1) a nonlinear Volterra integral equation arises, while is a Hammerstein equation in the case (2) [2].

Here we propose suitable Nyström methods in order to solve the nonlinear Hammerstein equation equivalent to the terminal value problem (2), using the tools of the polynomial approximation. In other words, suitable quadrature rules of product type, based on polynomial approximation of the smooth part appearing under the integral and exactly integrating the singularities, are proposed in order to construct the Nyström methods. Stability and convergence of the proposed methods are proved.

References

- [1] Diethelm K., Uhlig F. (2023) A New Approach to Shooting Methods for Terminal Value Problems of Fractional Differential Equations. *Journal of Scientific Computing*, vol. 97, article 38.
- [2] Diethelm K., Ford N. (2002) Analysis of Fractional Differential Equations. *Journal of Mathematical Analysis and Applications*, vol. 265(2), pp. 229–248.

Smooth Graph Signal Interpolation by Graph Basis Functions

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Keywords. Graph basis functions (GBF); kernel-based interpolation; global regularity.

Given a graph signal defined on a fixed number $M < N$, of vertices of a weighted graph with N vertices, and a positive definite graph basis function f , the interpolation procedure developed in [1] provides a graph signal that minimizes the kernel-dependent norm $\|\cdot\|_{K_f}$ which makes $(\mathbb{R}^N, \|\cdot\|_{K_f})$ a reproducing kernel space. Of course the interpolant depends on the chosen function f .

We will discuss the global regularity of the interpolant, as the positive definite graph basis function varies, using the notion of global regularity of a signal on a weighted graph introduced in [2].

Acknowledgements.

This research was supported by the Italian MUR project: Progetto di ricerca di rilevante interesse nazionale (PRIN 2022) 20227TRY8H – CUP J53D23003630001 – TIGRECO.

References

- [1] Erb W. (2022), Graph signal interpolation with positive definite graph basis functions. *Appl. Comput. Harmon. Anal.*, vol. 60, pp. 368–395.
- [2] Ricaud B., Shuman D.I, Vandergheynst P. (2013) On the sparsity of wavelet coefficients for signals on graphs. In: *Wavelets and Sparsity XV*, vol. 8858 (ed. by Van De Ville D., Goyal V. K., Papadakis M.), San Diego, California, USA.

Data-Driven Identification of Stochastic Differential Equations

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Keywords. Data-driven; sparse identification; SDEs.

Sparse Identification of Nonlinear Dynamics (SINDy) has recently emerged as an efficient data-driven method for discovering governing equations directly from time-series data, leading to simple and sparse models in a dictionary of candidate basis functions. The present work shows some aspects related to the implementation of this algorithm for the identification of the RHS of Stochastic Differential Equations (SDEs) and Stochastic Delay Differential Equations (SDDEs) with a single discrete delay. In particular, we show an approach which generates synthetic paths from a single observed trajectory via Euler-Maruyama simulations. The reference drift and diffusion terms are then reconstructed through proper estimation strategies belonging to Itô-Taylor expansions. We analyze the performance through RMSE on reconstruction accuracy of drift and diffusion estimates, together with CPU time with different data availability scenarios.

Acknowledgements. This research activity falls within the activities of PRIN-MUR 2022 project “Stochastic numerical modelling for sustainable innovation”, CUP: E53D23017940001, granted by the Italian Ministry of University and Research within the framework of the Call relating to the scrolling of the final rankings of the PRIN 2022 call.

References

- [1] Breda D., Conte D., D'Ambrosio R., Santaniello I., Tanveer M. (2026) Sparse Identification of Nonlinear Dynamics for Stochastic Delay Differential Equations. *J. Comput. Appl. Math.*, vol. 479, 117247.
- [2] Breda D., Conte D., D'Ambrosio R., Santaniello I., Tanveer M. (*Submitted*) A Matlab code for discovering governing dynamics in Stochastic Differential Equations using SINDy algorithm and Itô-Taylor based approximation.
- [3] Brunton S. L., Proctor J. L., Kutz J. N. (2016) Discovering governing equations from data by sparse identification of nonlinear dynamical systems. *PNAS*, vol. 113, pp. 3932–3937.
- [4] Wanner M., Mezić I. (2024) On higher order drift and diffusion estimates for stochastic SINDy. arXiv:2306.17814.

Early Warning System for Urban Pluvial Flooding Using Iber-SWMM

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Keywords. Early warning system; high performance computing; Iber.

Early Warning Systems (EWS) are essential tools for predicting and managing urban flooding [1]. This work presents a novel EWS for urban pluvial flooding based on the dual Iber-SWMM model [2], which integrates distributed hydrology, surface hydraulics and sewer network simulation, enabling bidirectional interactions between surface runoff and drainage systems.

The system incorporates high-resolution rainfall forecasts and is GPU-parallelized to ensure real-time operational efficiency. As a pilot case, the EWS was implemented in Sada, a small coastal municipality in Galicia, Spain, where recurrent flooding affects both urban and industrial areas. The system generates detailed predictions of water depths and flows across the urban domain, providing reliable flood forecasts and supporting timely flood risk management. This study demonstrates the potential of combining advanced hydrological-hydraulic modeling, high-resolution meteorological forecasts, and high-performance computing to anticipate localized flood impacts in complex urban environments.

Acknowledgements.

The work performed by Esteban Sanudo was funded by the Postdoctoral Fellowship Programme of the Xunta de Galicia (Conselleria de Educacion, Ciencia, Universidades e Formacion Profesional) [Ref. ED481B-2025/090].

References

- [1] Costabile P., Costanzo C., Kalogiros J., Bellos V. (2023) Toward street-level nowcasting of flash floods impacts based on HPC hydrodynamic modeling at the watershed scale and high-resolution weather radar data. *Water Resources Research*, vol. 59(10), article e2023WR034599. DOI: <https://doi.org/10.1029/2023WR034599>
- [2] Sanudo E., Garcia-Feal O., Hagen L., Cea L., Puertas J., Montalvo C, Alvarado-Vicencio R., Hofmann J. (2025) IberSWMM+: A high-performance computing solver for 2D-1D pluvial flood modelling in urban environments. *Journal of Hydrology*, vol. 651, article 132603. DOI: <https://doi.org/10.1016/j.jhydrol.2024.132603>

A Shallow Water Equation Based Model for Coastal Erosion Processes

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Keywords. Coastal erosion; shallow water equations; numerical model.

Coastal erosion is a natural process that alters the morphology of coastlines over time, causing a gradual loss of land area. In recent decades, this phenomenon has also been accelerated by human activities and climate change, with significant environmental, economic, and social consequences.

This work presents a numerical model for the analysis of coastal morphodynamic processes. The study integrates hydrodynamic and sediment transport processes through the implementation of a coupled model in COMSOL Multiphysics, based on the shallow water equations [1]. The model includes several relevant physical processes, including bottom friction, dispersion, submerged vegetation, and sediment transport. In particular, the effect of *Posidonia oceanica* meadows is modeled through a Morrison-type formulation [2], allowing the representation of wave energy dissipation induced by vegetation.

The results highlight the role of dissipative and dispersive terms in controlling wave propagation and ensuring the numerical stability of the solution. The presence of vegetation leads to a significant attenuation of wave energy, while sediment transport introduces a dynamic coupling with seabed morphology.

References

- [1] Lencina I. V. (2007) *Comparison Between 1D and 2D Models to Analyze the Dam Break Wave Using the FEM Method and the Shallow Water Equations*. Master's Thesis, Institute of Technology (KTH), Stockholm, Sweden.
- [2] Phana K. L., Stivea M. J. F., Zijlemaa M., Truonga H. S., Aarninkhofa G. J. (2019) The effects of wave non-linearity on wave attenuation by vegetation. *Coast. Eng.*, vol. 147, pp. 63–74.

Exploring Quasi-Variational Problems: Beyond the Classical Approaches

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Keywords. Variational analysis; set-valued analysis; normal cone operators.

Variational inequality theory has its origins with Stampacchia and Fichera and it provides a broad unifying setting for the study of optimization, complementarity problems and more in general equilibrium problems with the related applications in economics, finance, energy and game theory. In 1982, Chan and Pang [4] introduced the quasi-variational inequality: it is a variational inequality where the constraint set is subject to modifications depending on the considered point. The formulation of a quasi-variational inequality opportunely involves two set-valued maps: one whose values describe the constraint set and the other the principal operator of the problem. However, for some applications, the requirements of the classical results available in the literature could be too demanding. In order to address this fact and to reduce the gap between theory and applications, this talk proposes an overview on some recent developments [1–3] on the topic with the aim of going further.

References

- [1] Aussel D., Giuli M., Milasi M., Scopelliti D. (2025) A variational approach to weakly continuous relations in Banach spaces. *SIAM J. Optim.*, vol. 35, pp. 767–788.
- [2] Aussel D., Pezzola A., Riccardi R., Scopelliti D. (2025) On an alternative to convexity of the values in set-valued analysis. *Submitted*, pp. 1–20.
- [3] Castellani M., Giuli M., Milasi M., Scopelliti D. (2024) Projected solutions of generalized quasivariational problems in Banach spaces. *Nonlinear Anal. Real World Appl.*, vol. 76, pp. 1–12.
- [4] Chan D., Pang J.S. (1982) The generalized Quasi variational inequality problem. *Math. Oper. Res.*, vol. 7, pp. 211–222.

A New First-Order Proximal Method for Nonconvex and Nonsmooth Multilevel Optimization Problems

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Keywords. Multilevel optimization; nonconvex optimization; nonsmooth optimization; matrix factorization.

Many optimization models in applied mathematics involve nested structures and nonsmooth regularization terms, leading to objective functions that are both nonconvex and nondifferentiable. These features pose significant challenges for the design of efficient numerical algorithms with reliable descent properties.

In this work, we propose a first-order method for solving structured problems of this type. The scheme combines proximal constructions with block-wise proximal-gradient updates. At each iteration, updates associated with different components of the objective are computed via proximal-gradient mappings and then combined through a calibrated convex combination. The combination weights are selected to enforce the simultaneous decrease of suitable objective surrogates. The analysis relies on tools from variational analysis to characterize descent properties of the corresponding proximal-gradient mappings and to establish the existence of admissible descent directions. Under suitable conditions, the proposed scheme guarantees a monotonic decrease of the global objective functions despite the presence of nonconvex and nonsmooth terms.

We illustrate the approach on structured matrix factorization models with sparsity-promoting and low-rank inducing regularization, which arise in several contexts of interest in numerical analysis, including inverse problems, and signal processing.

References

- [1] Selicato L., Esposito F., Ang A. (2025) Binno: A 1st-order method for Bi-level Nonconvex Nonsmooth Optimization for Matrix Factorizations. ArXiv preprint *arXiv:2510.21390*.
- [2] Shafiei A., Kungurtsev V., Marecek J. (2024) Trilevel and multilevel optimization using monotone operator theory. *Mathematical Methods of Operations Research*. vol. 99, pp. 77–114.
- [3] Merchav R., Sabach S. (2023) Convex bi-level optimization problems with nonsmooth outer objective function. *SIAM Journal on Optimization*, vol. 33, pp. 3114–3142.

Multi-Agent System for Robust Optimization of Park & Ride Hubs

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Keywords. Multi-agent system; robust optimization; facility location; park & ride hubs.

Park & Ride (P&R) hubs are an important part of modern urban transportation systems, enabling travelers to change from private vehicles to public or ecological transport and thereby helping to reduce congestion and environmental impacts. Selecting optimal sites for new hubs from a predefined set of location candidates requires considering factors such as competition with existing hubs and accessibility, which influence the overall optimization process. While hub attractiveness depends on various factors such as distance, accessibility, and facility quality, a major challenge in P&R hub optimization arises from uncertainty in traveler behavior when choosing the most attractive hub.

This research focuses on the development of a Multi-Agent System (MAS) that identifies robust solution(s) to the above facility location problem for P&R hubs. Different agents are responsible for different customer behavior models that describe travelers' behavior when choosing a desired hub, while a base agent coordinates their communication in order to identify solution(s) that are acceptable to all customer behavior models involved in the process.

Each customer behavior agent applies artificial intelligence techniques to rank location candidates and identify promising ones by a particular customer behavior model. This knowledge enables the agent to propose solutions to be considered as robust solutions, taking into account different customer behavior models.

The proposed MAS has been experimentally investigated by solving different instances of P&R optimization problem using real data of Vilnius city and regions in Lithuania. The results of the investigation showed that the proposed MAS is able to identify sets of Pareto-optimal solutions for all test instances. The knee-point method enables the MAS to propose a decision on which solution can be considered as the robust one.

Acknowledgements.

This research has received funding from the Research Council of Lithuania (LMTLT), agreement No S-ITP-24-8, project title: Robust Optimization for Green Infrastructure Design using Artificial Intelligence (ROGIDAI).

Formal Methods and AI-Enhanced Modelling for Mathematics Education: The FMDV Approach

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Keywords. Formal methods; AI-driven modelling; mathematics education.

The growing integration of Artificial Intelligence (AI)-based tools is transforming the teaching and learning of mathematics and STEM subjects, reshaping processes of representation, modelling, and conceptual validation.

Research indicates that dynamic visualisations enhance conceptual understanding and covariational reasoning [4], while computational modelling supports the development of intuitions and pre-formal lines of reasoning that prepare students for more rigorous formalisation [1]. At the same time, AI expands pedagogical opportunities by improving feedback mechanisms, enabling personalised learning pathways, and supporting the multimodal construction of mathematical meaning [3], although its adoption requires responsible and pedagogically grounded integration [2].

This paper introduces the FMDV framework (Formal Methods - Modelling - Dynamic Visualisation) as a theoretical-operational model designed to coherently integrate mathematical formalisation, computational modelling, and AI-driven tools. The framework offers a structured and methodologically robust approach for innovating mathematics teaching practices in upper secondary education, supporting the transition from exploratory reasoning to formal understanding within contemporary STEM learning environments.

References

- [1] Borwein J., Bailey D. (2008) *Mathematics by experiment*. Boca Raton: CRC Press.
- [2] Walkington C. (2025) The implications of generative artificial intelligence for mathematics education. *School Science and Mathematics*, pp. 1–10.
- [3] Yeo S., Moon J, Kim D. (2024) Transforming mathematics education with AI: Innovations, implementations, and insights. *Mathematics Education*, 63, pp. 386–392.
- [4] Zhang Y., Wang P., Jia W., Zhang A., Chen G. (2025) Dynamic visualization by GeoGebra for mathematics learning: a meta-analysis of 20 years of research. *Journal of Research on Technology in Education*, 57(2), pp. 437–458.

VarUnicoding: a GP-based Machine Learning for Multivariable Multidimensional Function Search

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Keywords. Genetic programming; unsupervised machine learning; symbolic regression, function search; control.

Genetic Programming methods are gaining increasing popularity in problems where an unknown function must be found. The problem can have training data, as an approximation, and various neural networks perform here well. However, unsupervised machine learning can often be a problem in many engineering or physical areas, and here GP offers significant potential advantages. GP outperforms neural networks in scenarios with small or no data or requiring analytical form. But most existing GP-based methods are capable of finding one or two-dimensional functions with a small number of arguments due to the complexity of non-numerical optimization space. In practical applications, multidimensional functions of many variables must be found. This paper proposes a new GP-based method, VarUnicoding, which can find functions in such a complex spaces in a reasonable amount of time. The paper describes a new encoding procedure, the implementation of genetic operations, and the principle of small variations, which enables the method to handle multidimensional search problems. A practical numerical example of solving a quadcopter control function search problem is presented to demonstrate the effectiveness of the method.

Acknowledgements.

This research was supported by the Russian Science Foundation under grant No 25-21-00704.

References

- [1] Diveev A. I., Shmalko E. Yu., Sofronova E. A. (2018) Multipoint numerical criterion for manifolds to guarantee attractor properties in the problem of synergetic control design. *ITM Web of Conferences*, vol. 18, p. 01001.
- [2] Kolmogorov A. N., Fomin S. V. (1957) *Elements of the Theory of Functions and Functional Analysis*. Volume 1. Metric and Normed Spaces. Graylock Press. Rochester, N.Y. (130 pages).
- [3] Mizhidon A. D. (2011) On a Problem of Analytic Design of an Optimal Controller. *Automation and Remote Control*, vol. 72(11), pp. 2315–2327. Doi: 10.1134/S0005117911110063

Analysis of Data Imputation Methods for Missing Economic Indicator Values at NUTS 2 Level Data Set

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Keywords. Machine learning; missing values; economic indicators; imputation methods comparison.

This article analyzes machine learning and statistical methods for imputing missing values and compares metrics for model prediction accuracy.

The aim of this study is to select the most appropriate method for imputing missing values in the NUS2-level dataset of economic indicators for EU regions. The Random Forest and XGBoost methods used in previous studies did not achieve the required accuracy because the missing values in this dataset are not random; they exhibit internal dependencies on time, region, and country. Furthermore, when analyzing the dataset collected from Eurostat, it was noticed that there are often missing data in regions at the beginning and end of the period, or that data for certain economic indicators are missing entirely.

During the study, a methodology was developed for data input, in which the average of neighbouring values was applied to individual random missing values. For values missing at the beginning and/or end of the regional period, linear, quadratic, cubic, exponential, and logarithmic forecasting methods were applied, and MAPE errors were calculated. The smallest MAPE errors and the most accurate forecast values for each region were selected for each indicator. This dataset, supplemented with missing values, will be used to forecast regional resilience.

Acknowledgements. This research has received funding from the Research Council of Lithuania (LMTLT), agreement No. S-MIP-24-35.

References

- [1] Botchkarev A. (2019) A New Typology Design of Performance Metrics to Error Measures in Machine Learning Regression Algorithms. *Interdiscip. J. Inf. Knowl. Manag.*, vol. 14, pp. 45–76. DOI: 10.28945/4184.
- [2] Monir Aljinbaz A. M., Mahmoud M., Rahhal A. (2024) Forecasting Unemployment Rate for Multiple Countries Using a New Method for Data Structuring. *IJACSA*, vol. 15(12). DOI: 10.14569/IJACSA.2024.0151205.
- [3] Sambasivam D., Ilakiyaselvan N. (2023) A Machine Learning Approach to Predict Economic Freedom Index. *IJAR SCT*, vol. 3(1), pp. 486–496. DOI: 10.48175/IJAR SCT-12986.

Causal Inference Analysis on Dengue Interventions in Baguio City Philippines from 2010 to 2022

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Keywords. Causal Inference; Machine Learning; Dengue Intervention.

Causal inference can be used to analyze the relationship between variables in a system. Causality aims to uncover the true cause and effect from data that cannot be described by correlation. Dengue continues to infect the human population. In Baguio City, Philippines, dengue cases reached 1,500 cases in 2025. The government is consistently implementing various interventions to eradicate these cases. In this research, we investigate the effectivity of these interventions through causal inference. For the period 2010 to 2022, seven interventions were implemented to address dengue infection in the smallest government units of Baguio City, called barangays. These interventions were applied to Irisan and Bakakeng Central barangays which are considered hotspots for having consistently high dengue infections. Initial results indicate that the first intervention was successful for both barangays. The third and sixth interventions were successful for barangay Irisan while the second intervention was successful for barangay Bakakeng Central. Result of this research can serve as baseline values and historical reference for future policy making in intervention implementation for dengue infections.

References

- [1] City Government of Baguio, “Baguio Records 7 Deaths Due to Dengue; Cases Up 462%.”: <https://new.baguio.gov.ph/news/baguio-records-7-deaths-due-to-dengue>, accessed March 13, 2026.
- [2] Department of Health. “Health Department Promotes Dengue Awareness.” ReliefWeb, 18 June 2013, <https://reliefweb.int/report/philippines/health-department-promotes-dengue-awareness>, accessed 14 March, 2026.
- [3] Ayo S.K. (2023) Exploring the Power of Causal Inference in Machine Learning. <https://albusdd.medium.com/exploring-the-power-of-causal-inference-in-machine-learning-522467960daa>, Medium, accessed March 19, 2026.
- [4] City Health Services Office - Baguio. (2014, August 7). Dengue cases down in Baguio; health office reminds public to remain vigilant. ReliefWeb. <https://reliefweb.int/report/philippines/dengue-cases-down-baguio-health-office-reminds-public-remain-vigilant>, accessed March 13, 2026.

Weak Shock Diffraction-Reflection Analysis for a Rigid Wedge in Extended Chaplygin Gas

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Keywords. Weak shock reflection; R-H conditions; asymptotic expansion; weakly nonlinear geometric acoustics.

This research examines the diffraction and reflection processes that occur when a weak shock wave interacts with a wedge inclined at an arbitrary angle, within the framework of a more realistic extended Chaplygin gas model. Asymptotic solutions of the two-dimensional Euler equations are derived systematically, together with appropriate boundary conditions that characterize the diffraction of a weak shock as it encounters the wedge. The formulation incorporates the distinctive thermodynamic properties of the extended Chaplygin gas, enabling a detailed assessment of how these properties influence the resulting flow configuration. Particular attention is given to the analysis of the local flow structure near a singular point, highlighting the significant role of the gas behavior in determining the overall dynamics of the flow. In addition, a comparative study is carried out to illustrate the differences between the effects of a polytropic gas and those of an extended Chaplygin gas on the global flow structure.

References

- [1] Gupta N., Sharma V.D. (2015) On shock reflection–diffraction in a van der Waals Gas. *Studies in Applied Mathematics*, vol. 135(2): pp. 171–195.
- [2] Chen G. Q., Feldman M. (2010) Global solutions of shock reflection by large-angle wedges for potential flow. *Annals of Mathematics*, pp. 1067–1182.
- [3] Upadhyay G., Singh A., Singh L. P. (2026) Two-dimensional analysis of weak shock diffraction and reflection in extended Chaplygin gas. *Physics of Fluids*, vol. 37(10): article 106132.

A Comparative Analysis of Three Infinity Methodologies

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Keywords. Cantor; Robinson, grossone; infinities and infinitesimals.

In this talk, I continue the study of infinities and infinitesimals started in [4]. I propose a comparative table that highlights selected peculiarities and major distinctions among the approaches to infinity and infinitesimals proposed by Cantor, Robinson, and Sergeyev (see [1–3], respectively). The aspects chosen for comparison are intended to be accessible not only to specialists working within each of the three frameworks, but also to a broader audience. Questions concerning internal technical problems of each theory have been deliberately set aside. Instead, the comparison emphasizes features that are either foundational – thus transversal across the frameworks – or practically relevant for applications. The table is designed to assist mathematicians, computer scientists, philosophers, and engineers in identifying which methodology may be most appropriate for their specific problems.

The performed analysis makes clear that all three methodologies have their own strengths and limitations, and that each can be effectively employed depending on the nature of the research problems under investigation. The principal distinction of the grossone-based methodology proposed in [3] lies in its integration of highly unconventional foundational principles (such as new ways for measuring infinite sets) with practical numerical computations. In particular, it operates with numerical entities that may contain finite, infinite, and infinitesimal components, whereas the preceding methodologies remain purely theoretical in nature.

References

- [1] Cantor G. (1955) *Contributions to the founding of the theory of transfinite numbers*. Dover Publications, New York.
- [2] Robinson, A. (1996) *Non-standard Analysis*, Revised edn., Princeton University Press, Princeton, NJ.
- [3] Sergeyev, Ya. D. (2017) Numerical infinities and infinitesimals: Methodology, applications, and repercussions on two Hilbert problems. *EMS Surveys in Mathematical Sciences*, 4(2), 219–320.
- [4] Sochkov, A.L. (2009) Philosophical aspects of the newest arithmetic of infinity. *Vestnik of Lobachevsky State University of Nizhni Novgorod. Series: Social Sciences* **3(15)**, 72–77 (in Russian).

Machine Learning for Optimal Traffic Lights Settings in Changing Environment

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Keywords. Traffic control; machine learning; symbolic regression; traffic flow model.

Traffic flow control using traffic light regulation at intersections is considered. A recurrent finite-difference model for traffic flow control, built on the basis of controlled network theory, is used. The traffic flow is controlled by switching the phases of the traffic lights at intersections, which deletes or creates certain edges of the network graph, corresponding to the permission or prohibition of certain maneuvers. The result of solving the optimal control problem is the optimal coordination plan for the traffic lights.

The paper focuses on the situation when there is a sharp change in traffic flow. A sharp change in traffic flow can be caused by traffic accidents, road repairs, etc. Then the optimal coordination plan calculated is no longer optimal, and traffic engineers have to make certain decisions to resolve the issue. It takes their expertise, time, and costs. In such cases, it is proposed to design a control function that adjusts the duration of traffic light phases based on the difference between the actual traffic flow and the flow used for the calculated coordination plan. The control function is designed or synthesized in advance together with the coordination plans based on the data obtained from the model. It assists the traffic engineer in managing difficult situations. The paper formulates the problem of traffic flow control synthesis. To solve the problem, machine learning using symbolic regression methods is proposed. The result of solving the synthesis problem is a control function that changes the duration of the traffic light phases depending on the traffic flow in the sections of the network. An example of a solution of the traffic flow control synthesis problem is given for a group of intersections in Moscow.

References

- [1] Sofronova E. A., Diveev A. I. (2024) Signal Timing Optimization by VarGA: Case Study. *2024 IEEE Intelligent Vehicles Symposium (IV)*, Jeju Island (Republic of Korea), pp. 3120–3125.
- [2] Sofronova E. A., Diveev A. I. (2020) Traffic flows optimal control problem with full information. In: *Proc. IEEE 23rd Int. Conf. Intell. Transp. Syst.*, pp. 1–6.

Path-Following Control Law Distillation by Symbolic Regression

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Keywords. Path-following; optimal control; machine learning; symbolic regression.

To implement the solution of the optimal control problem (OCP) directly in the control system of a real object operating in a changing environment, a motion stabilization system along a given trajectory or path is required. Path-following is successfully applied in the cases where it is necessary to accurately cover the path rather than keep the desired speed. Reviews of the main path-following control strategies for different types of autonomous robotic vehicles often use nonlinear control techniques such as backstepping, vector field, model predictive control (MPC), as well as reinforcement learning-based control, symbolic regression (SR), etc. In this paper, symbolic regression methods [1] are used to automatically construct the motion stabilization system as an interpretable mathematical expression. The practical feasibility of the solution obtained is tested experimentally. This verification is assumed to be sufficient at the control systems development stage.

Another challenge of the OCP considered is the movement along a loop trajectory. The goal is to maneuver around obstacles and then return to the initial state. Such conditions arise when calculating the optimal trajectory along a circuit, for example, in Formula 1 racing or F 1/tenth mobile robot racing of 1:10 scaled autonomous race cars. To solve the problem, mandatory passage domains or gates are placed in the state space. The performance criterion encompasses not only the primary performance one, but also penalties for colliding with obstacles and for failing to enter the mandatory passage gates with the certain accuracy. To address the classical OCP with complex phase constraints, a direct approach [2] is employed, which involves transforming the problem into a finite-dimensional optimization problem and implementing evolutionary techniques.

References

- [1] Udrescu S.-M., Tegmark M. (2020) AI Feynman: A Physics-Inspired Method for Symbolic Regression. *Science Advances*, vol. 6(16), eaay2631.
- [2] Bertolazzi E., Biral, F. (2025) A Direct/Indirect Approach to Optimal Control Problems. *LNCS*, volume 14477, 4th International Conference NUMTA 2023, pp. 47–62.

Convergence Analysis for Nonlinear Boundary Value Problems with the Riesz-Caputo Operator

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Keywords. Fractional calculus; Riesz-Caputo derivative; B-splines.

Fractional boundary value problems are instrumental in modeling complex systems characterized by memory effects and anomalous diffusion. This work focuses on nonlinear fractional boundary value problems governed by the Riesz-Caputo operator, a framework particularly well-suited for capturing symmetric diffusive phenomena. We first establish the well-posedness of the problem by providing an integral representation of the solution, thereby ensuring its existence and uniqueness. Subsequently, we introduce a robust B-spline collocation method designed to efficiently approximate the solution and finally present a comprehensive convergence analysis, supported by both rigorous theoretical proofs and validating numerical experiments.

References

- [1] Sorgentone C., Pellegrino E., Pitolli F. (2025) A spline-based framework for solving the space-time fractional convection-diffusion problem. *Mathematics Letters*, vol. 161, article 109370.

Least-Squares Shooting with Runge-Kutta Substepping for DAE BVPs

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Keywords. Boundary value problems; differential-algebraic equations; least-squares; shooting, Runge-Kutta methods.

BVPs for DAEs arise in several scientific and engineering applications. When shooting methods are employed, the sensitivities are propagated along the numerical integration of the DAE system. However, in regions of the solution mesh with stiff dynamics, the use of large integration steps may lead to poor convergence of the nonlinear solver due to inaccurate sensitivity information [1]. Another challenge in the numerical solution of BVP for DAEs is the presence of a constraint manifold that must be satisfied at each point of the solution mesh. If the solution method does not accurately capture this manifold—either via a high-order implicit scheme or through index reduction—numerical instabilities may arise [2].

In this work we propose a shooting method for BVPs combined with a local time substepping. In this scheme the states and the associated Jacobians are propagated by differentiating the discrete RK algorithm. When the dynamics exhibit stiff behavior, the integration step can be subdivided into a prescribed number of substeps, allowing a more reliable propagation of sensitivities without modifying the global mesh and thus without increasing the dimension of the nonlinear system to be solved. Moreover, we also propose a least-squares formulation of the shooting method, which enables us to handle the constraint manifold along the solution mesh in a robust manner and to enable a simple solution of BVPs for DAE systems.

Numerical experiments demonstrate that the use of least-squares shooting and local time substepping improves the robustness of the solver in the presence of stiff dynamics described by DAEs, while maintaining computational efficiency.

Acknowledgements. This study was carried out within the project “*SPACE IT UP! ASI contract n. 2024-5-E.0 CUP Master n. I53D24000060005*” funded by the Italian Space Agency (ASI), and the Ministry of University and Research.

References

- [1] März R. (1984) On difference and shooting methods for boundary value problems in differential-algebraic equations *Z. Angew. Math. Mech.*, vol. 64, pp. 463–473.
- [2] Amodio P., Mazzia F. (1993) Boundary value methods for the solution of differential-algebraic equations. *Numer. Math.*, vol. 66, pp. 411–421.

Fast Parallel Computation of Numerical Solutions for ODEs Using the IMT-DE Transformation

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Keywords. Variable transformation; parallel computing; multi-GPU.

This paper considers numerical methods for parallel computation of ordinary differential equations. The proposed method is based on the numerical solver for initial value problem of ordinary differential equations by the IMT-DE type numerical indefinite integration formula [2].

The problem is transformed into an equivalent integral equation and solved using Picard iteration. In this formulation, the solution over the entire time interval is obtained simultaneously. This structure naturally leads to time-parallel computation and makes the method well suited for parallel implementation.

The integral operator in the iteration is approximated using the IMT-DE numerical indefinite integration formula [1]. To accelerate the computation, we implement the solver on a multi-GPU architecture and employ Halo exchange for efficient communication between GPUs.

Numerical experiments demonstrate that the proposed method achieves computational speedup compared to the conventional parallel-in-time method, the Parareal method, especially in larger-scale calculations.

Acknowledgements.

This research was supported by JSPS KAKENHI Grant Number 24K06840.

References

- [1] Ogata H. (2022) Numerical indefinite integration using the IMT-type DE transform and the Sinc approximation for periodic functions *Transactions of the Japan Society for Industrial and Applied Mathematics*, vol. 32, pp. 1–19.
- [2] Ogata H. (2023) Numerical solver for initial value problems of ordinary differential equations by the IMT-DE type numerical indefinite integration formula *Transactions of the Japan Society for Industrial and Applied Mathematics*, vol. 33, pp. 36–65.

Grossone and the Continuum Problem

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Keywords. Grossone; continuum problem; infinite sets; real numbers.

The approach to the infinite fostered by the theory of grossone and its multiple computational applications (see [3]) suggest a host of considerations relating to the Continuum Problem (see [1, 2, 4]). In this talk, first we briefly review fundamental facts about:

1. The theory of grossone, through which one may assign an Euclidean measure to certain infinite sets, in particular, to the set of natural, odd, even, and integer numbers.
2. The problem of measuring the (size of the) continuum.
3. The Continuum Problem.

Then, we address the questions of how the grossone-theoretic measures of the continuum (better, of different continua) and, generally speaking, the grossone-theoretic methodology reveal aspects of the complexity and intractability of the Continuum Problem construed in the traditional (set-theoretic) sense.

References

- [1] Cantor G. (1955) *Contributions to the founding of the theory of transfinite numbers*. Dover Publications, New York.
- [2] Sergeyev Y. D. (2010) Counting systems and the First Hilbert problem. *Nonlinear Analysis: Theory, Methods & Applications*, vol. 72(3-4), pp. 1701–1708.
- [3] Sergeyev Y. D. (2017) Numerical infinities and infinitesimals: Methodology, applications, and repercussions on two Hilbert problems. *EMS Surveys in Mathematical Sciences*, vol. 4(2), pp. 219–320.
- [4] Woodin W. H. (2001) The Continuum Hypothesis, Part I. *Notices of the American Mathematical Society*, vol. 48(6), pp. 567–576.

Modelling Human Skin Morphology and Simulating Transdermal Transport of 50 Chemicals

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Keywords. Transdermal delivery; dermal uptake; finite dose; numerical simulation and computational modelling; homogenized diffusion model; diffusion coefficient; partition coefficient; simulation of 50 chemicals.

A mathematical model was developed to investigate the transdermal transport of 50 chemicals across human skin. The model incorporates key skin properties and physicochemical parameters. Simulations were performed using experimentally determined partition and diffusion coefficients for the different skin layers. The predicted uptake dynamics reproduced the characteristic temporal trends reported in the experimental studies. The simulation results demonstrate that the reduced-complexity mesoscale diffusion model captures essential temporal features of chemical uptake and allows comparison of uptake behaviour between chemicals. Future work will address additional physiological and experimental factors to improve quantitative predictive accuracy.

Acknowledgements.

This research was supported by King Abdullah University of Science and Technology (KAUST).

Noise-Aware Preference Optimization for Large Language Models

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Keywords. AI; machine learning; preference optimization; LLM.

Preference-based optimization is an important framework for improving the behavior of large language models. In particular, Direct Preference Optimization (DPO) [1] provides an efficient alternative to reinforcement learning from human feedback by casting preference learning as a tractable optimization problem. However, when preferences are produced by AI systems rather than human annotators, the resulting labels may contain substantial noise that reduces the final model quality.

In this work, we study robust preference optimization under noisy supervision and propose two extensions of DPO: noise-robust DPO (nrDPO) and nrDPO-gated. The first method introduces a weighted objective in which each preference pair is scaled according to three factors: reference-margin confidence, context stability, and length-based correction. This yields a simple and computationally efficient scheme for down-weighting unreliable training pairs. The second method augments this objective with a gating rule that filters sign-inconsistent pairs using the reference-policy margin.

The proposed methods are evaluated on the Anthropic HH-RLHF benchmark [2]. The experiments show that AI relabeling may invert more than half of the original human preferences, creating a challenging optimization landscape. Under these conditions, standard DPO is sensitive to label corruption, while the proposed robust variants show more stable behavior. In particular, nrDPO-gated improves preference accuracy in high-noise regimes and recovers competitive performance in realistic reinforcement learning from AI feedback scenarios.

Acknowledgements.

This research was funded by the Committee of Science of the Ministry of Science and Higher Education of the Republic of Kazakhstan under grant number AP23489782.

References

- [1] Rafailov R. et al. (2023) Direct preference optimization: your language model is secretly a reward model. In: *NIPS '23*. Curran Associates Inc., Red Hook, NY, USA, pp. 53728–53741.
- [2] Anthropic (2022) HH-RLHF: Helpful and harmless human preference dataset.

Mathematical Analysis of Weak Shock Diffraction in a Realistic Extended Chaplygin Gas

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Keywords. Weak shock reflection; Chaplygin gas; R-H conditions; asymptotic expansion; weakly nonlinear geometric acoustics.

The study investigates the diffraction and reflection of a weak shock interacting with a right-angled wedge for a realistic extended Chaplygin gas. Asymptotic solutions of the two-dimensional Euler equations are derived under suitable boundary conditions that describe the diffraction of a weak shock by the wedge. The influence of the chosen gas model is incorporated into the formulation, allowing its effects on the resulting flow configuration to be examined in detail. Graphical representations of the diffracted shock, expansion waves, and shock locations are presented for various relevant parameters. In particular, the local flow structure near the singular point is analyzed thoroughly, emphasizing the important role played by the properties of the extended Chaplygin gas in determining the overall flow behavior.

References

- [1] Hunter J. K., Keller J. B. (1984) Weak shock diffraction. *Wave Motion*, vol. 6(1), pp. 79–89.
- [2] Chen G. Q., Feldman M. (2010) Global solutions of shock reflection by large-angle wedges for potential flow. *Annals of Mathematics*, pp. 1067–1182.
- [3] Upadhyay G., Singh A., Singh L. P. (2025) Weak shock diffraction and reflection in extended Chaplygin gas. *Studies in Applied Mathematics*, vol. 156(1):e70178.

Low-Order Polynomial Models for Accurate Prediction of Paracetamol Release Kinetics

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Keywords. Surrogate modelling; polynomial regression; USP Apparatus 2.

Accurate prediction of drug dissolution profiles is critical for optimising oral solid dosage forms, yet traditional in vitro testing methods are often time-consuming and labour-intensive. This study investigates the application of surrogate modelling to efficiently approximate paracetamol dissolution kinetics under varying hydrodynamic conditions.

We employed a Design of Experiments (DoE) approach with two input factors: tablet off-centre position and impeller rotational speed (25 - 75 RPM), and selected four time points (5, 10, 20, and 30 minutes) for cumulative drug release. Two surrogate modelling techniques were evaluated: polynomial regression (up to third order) and Gaussian Process Regression (GPR). Model performance was rigorously validated using Leave-One-Out Cross-Validation, Lack-of-Fit F-tests, external validation, and goodness-of-fit metrics (R², RMSE).

Results indicate that polynomial models significantly outperformed GPR for this specific system. Specifically, a second-order polynomial provided the best fit for the initial dissolution phase (5 min), while third-order polynomials consistently minimised prediction errors for subsequent time points. The analysis revealed that while increased impeller speed universally accelerated release, the influence of tablet position became increasingly significant at higher rotational speeds, suggesting a strong hydrodynamic interaction. These findings demonstrate that low-order polynomial surrogates offer a computationally efficient and accurate alternative to complex stochastic models for predicting dissolution behaviour in standard USP Apparatus 2 configurations, potentially reducing experimental costs and accelerating process development.

Acknowledgements.

This research is funded by the Latvian Council of Science, project "Hierarchical model approach and experimental validation for typical drug discovery system dissolution profile predictions (ModelDrug)", project No lzp-2023/1-0078.

Unimaginable Numbers and Names within the LLM Explosion

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Keywords. Unimaginable numbers; LLM (Large Language Model); Grossone.

This talk investigates the critical limitations of current computational paradigms and proposes a strategic shift toward non-classical extensions of the number concept. As the “LLM explosion” continues to redefine the boundaries of artificial intelligence, the heavy reliance on traditional binary (0/1) logic faces significant hurdles in terms of energy efficiency, data security, and the accurate modeling of complex environments such as Reality+ and Virtual Reality (VR). We explore different mathematical extensions and their physical implications, viewing them as indispensable tools for optimizing performance within future digital and quantum realities.

A central focus of the presentation is the introduction of “new numbers” and their symbolic representations, specifically analyzing the Grossone methodology for infinite and infinitesimal computing and the Nondum notation for handling not-yet-known or approximate values. By revisiting their Pythagorean roots, we demonstrate how these frameworks offer a robust alternative to the brute-force scaling of current Large Models. The talk highlights how integrating unimaginable numbers and alternative numerical names can enhance computational security and sustainability, providing a more coherent mathematical foundation for the immersive and high-performance infrastructures of tomorrow.

References

- [1] Caldarola F., d’Atri G., Mercuri P., Talamanca V. (2020) On the arithmetic of Knuth’s powers and some computational results about their density. In: Sergeyev Y.D., Kvasov D.E. (eds.), *Numerical Computations: Theory and Algorithms 3rd Int. Conf.*, Springer, LNCS Vol. 11973, pp. 381–388.
- [2] Leonardis A., d’Atri G., Caldarola F. (2022) Beyond Knuth’s notation for unimaginable numbers within computational number theory. *International Electronic Journal of Algebra* Vol. 31, pp. 55–73.
- [3] Sergeyev Y. D. (2017) Numerical infinities and infinitesimals: methodology, applications, and repercussions on two Hilbert problems. *EMS Surv. Math. Sci.*, Vol. 4, pp. 219–320.

Segmentation of Medical and Material Images

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Keywords. Image segmentation; snake curves; mammography; microplastics.

Despite advances in image segmentation methods from materials science to biomedical imaging, the accurate contour extraction remains a complex task, especially in presence of high noise levels and low contrast [1].

In this framework, we compare several segmentation techniques, which include geometric active contours and snakes based on gradient vector flow [2], analyzing their performance in two applications. One is mass segmentation in X-ray mammograms, which may exhibit significant variability in gray level distribution depending on breast features and image post-processing. The other is morphological characterization of microplastics in optical microscopy images, which can contain aggregated particles difficult to distinguish. The segmentation methods are combined with different pre-processing strategies for contour extraction optimization. The analyzed mammograms are taken from open-access clinical databases, while the microscopy images are acquired on ad hoc prepared samples, containing plastic standards, with known size and shape, and irregularly shaped particles originated from contamination.

Acknowledgements.

This research was supported by projects 22HLT05 MAIBAI (European Partnership on Metrology), PNRR METROFOOD-IT, CRT “Tecniche spline per la costruzione di modelli anatomici digitali nello screening mammografico”.

References

- [1] Brar K. K., Goyal B., Dogra A., Mustafa M. A., Majumdar R., Alkhayyat A., Kukreja V. (2025) Image Segmentation Review: Theoretical Background and Recent Advances. *Information Fusion*, vol. 114, 102608.
- [2] Wang D., Dang X., Liu W., Wang Y. (2023) Image segmentation using active contours with image structure adaptive gradient vector flow external force. *Front. Appl. Math. Stat.*, vol. 9, 1271296.

Spectral Complexity of Deep Neural Networks

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Keywords. Deep learning; neural networks; isotropic random fields; Gaussian processes; compositional kernels; angular power spectrum; model complexity.

It is well-known that randomly initialized neural networks weakly converge to Gaussian processes as the width of all layers goes to infinity. In my talk, I will propose to use the spectrum of the limiting Gaussian process kernel to characterize the complexity of the network architecture. In particular, I will define sequences of random variables associated with the spectrum, and provide a full characterization of the network complexity in terms of the asymptotic distribution of these sequences as the depth diverges. On this basis, I will classify neural networks as low-disorder, sparse, and high-disorder. I will show how this classification highlights a number of distinct features for standard activation functions, and in particular sparsity properties for the ReLU.

This is joint work with Simmaco Di Lillo, Domenico Marinucci and Michele Salvi.

Acknowledgements.

This work was partially supported by the MUR Excellence Department Project MatMod@TOV awarded to the Department of Mathematics, University of Rome Tor Vergata, CUP E83C18000100006. We also acknowledge financial support from the MUR 2022 PRIN project GRAFIA, project code 202284Z9E4, the INdAM group GNAMPA and the PNRR CN1 High Performance Computing, Spoke 3.

References

- [1] Di Lillo S., Marinucci D., Salvi M., Vigogna S. (2025) Spectral complexity of deep neural networks. *SIAM Journal on Mathematics of Data Science*, vol. 7(3).
- [2] Di Lillo S., Marinucci D., Salvi M., Vigogna S. (2025) Fractal and Regular Geometry of Deep Neural Networks. [arXiv:2504.06250v1](#).

Continuity and Homogeneity in the Infinity of the Ancient World: Two Approaches to Incommensurability

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Keywords. Continuous infinite divisibility; homogeneity; incommensurability.

The notion of infinity in the ancient world had repercussions in both Greek and Mesopotamian culture. This paper aims to compare the mathematical idea of infinity in both cases. On the one hand, it is of interest not only to the contribution presented in Euclid's *Elements* but also to Aristotle's contribution in both mathematics and physics, given that this period saw a consolidation of this notion of infinity after several centuries of debate on the matter. With the exception of Archimedes of Syracuse (both because of the successful discovery of his results and because of its radically exogenous nature), the standard Aristotelian-Euclidean tradition within ancient Greece remained unchanged.

On the other hand, during the Old Babylonian period, the mathematics conceived there allowed for dealing with infinity not only in cases of exact answers but also through approximations in situations where exactness was impossible.

In this paper, we will argue that this distinction stems from two disparate methodological styles that appeal to mathematical tools such that Greek incommensurability arising from impossible operations with heterogeneous magnitudes, and in cases of equal dimension, demonstrate a narrow methodological approach in the Greek mainstream case. In contrast, Old Babylonian mathematics presents an opening in both types of incommensurability, offering successful approximations in cases where Greek standard mathematics remains speechless.

References

- [1] Euclid (1926) *The Thirteen Books of Euclid's Elements. Introduction and Notes by Sir Thomas L. Heath*. Cambridge University Press, Cambridge.
- [2] Hoyrup J. E. (2002) *Lengths, Widths and Surfaces. A Portrait of Old Babylonian Algebra and its Kin*. Sources and Studies in the History of Mathematics and Physical Sciences. Springer, New York.

Data-Driven Numerical Bifurcation Analysis of Partially Observed Multi-Pathogen Disease Dynamics

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Keywords. Multi-pathogen disease dynamics; numerical bifurcation analysis; data-driven modeling; partial observability.

Mathematical models are widely used to study the spread and interaction of multiple infectious diseases, classical numerical bifurcation analysis typically assumes that governing equations, parameters and system states are fully known. In practice, epidemiological systems are often partially observed, with incomplete or noisy and uncertain interaction mechanisms. This limits the applicability of standard bifurcation techniques. This creates a gap between theoretical bifurcation analysis and real-world multi-pathogen disease data.

In this work, a data-driven numerical framework is investigated for the analysis of bifurcation behavior in partially observed multi-pathogen disease systems. Rather than relying on fully specified mechanistic models, the approach aims to reconstruct reduced-order dynamical representations directly from available time-series data using sparse system identification techniques. Numerical continuation methods are then employed to explore how qualitative system behavior may change as key epidemiological parameters vary, with the objective of identifying stability changes and recurrent outbreak dynamics.

The study is being developed using illustrative multi-pathogen scenarios, including interacting infections such as tuberculosis and pneumonia, where partial observability is common. This work represents an initial step toward bridging data-driven inference and numerical bifurcation analysis in settings where system knowledge is incomplete, with the longer-term goal of supporting the systematic analysis of complex disease interactions under realistic data constraints.

References

- [1] Rosafalco L., Conti P., Manzoni A., Mariani S., Frangi A. (2025) Online learning in bifurcating dynamic systems via SINDy and Kalman filtering. *Nonlinear Dynamics*, vol. 113, pp. 14201–14221.
- [2] Dhooge A., Govaerts W., Kuznetsov Y.A., Meijer H.G.E., Sautois B. (2008) New features of the software MatCont for bifurcation analysis of dynamical systems. *Mathematical and Computer Modelling of Dynamical Systems*, vol. 14(2), pp. 147–175.

Data-Dependent Moving Least Squares and Applications

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Keywords. Non-linear approximation; WENO methods.

In this talk, we present a data-dependent modification of the moving least squares (MLS) method aimed at reducing the Gibbs phenomenon near discontinuities. Our approach introduces new weight functions that assign smaller weights to nodes located close to discontinuities, while preserving the decay with respect to the distance from the point of approximation. This strategy enables a sharper reconstruction of discontinuous data and mitigates spurious oscillations. A key component of the method is the use of smoothness indicators—borrowed from the data-dependent WENO framework—to identify nodes influenced by discontinuities. These indicators guide the construction of data-dependent weights that depend on both spatial distance and local smoothness.

In addition to modifying the weight functions, we investigate the role of the shape parameter in the approximation, adjusting it in a data-driven manner to further enhance accuracy in the vicinity of discontinuities. We analyze the resulting approximant in terms of polynomial reproduction, accuracy, and smoothness, and study its behavior with respect to diffusion and oscillatory artifacts. Furthermore, we extend the analysis beyond the classical ℓ^2 -based formulation of MLS by considering alternative norms, such as the ℓ^1 norm and other possible choices, in order to evaluate their impact on the reconstruction properties and robustness of the method. Numerical experiments support the theoretical findings and illustrate the advantages of the proposed modifications.

Acknowledgements. This research was supported by grant PID2023-146836NB-I00, funded by MCIN/AEI/10.13039/501100011033, and GVA project CIAICO/2024/089.

References

- [1] Levin D., Ramón J. M., Ruiz-Álvarez J., Yáñez D. F. (2025) Data-dependent moving least squares. *Appl. Math. Num.*, Vol. 216.

A Block-Based Filled Function Method for Global Optimization Problems

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Keywords. Global optimization; filled function method; block-based subspace decomposition.

Numerous complex problems in engineering and economics are modeled as global optimization problems. Because objective functions of these problems often contain multiple local minima, standard algorithms struggle to reach the global minimum. Auxiliary function techniques address this by penalizing discovered local minima and allowing the search process to escape local basins of attraction.

This study proposes a new Filled Function Method to reach the global minimum. Escaping local minima requires minimizing the constructed filled function, a process highly sensitive to initial points. To overcome the resulting computational bottleneck in high-dimensional problems, a block-based subspace decomposition strategy is integrated into the search process. Within this framework, the effects of initialization strategies such as Random Sampling, Latin Hypercube Sampling, Sobol Sequences, and Grid Search are systematically investigated. The computational efficiency and solution quality of the method are tested on various global optimization problems. Comparative numerical results demonstrate the theoretical and practical advantages of the proposed approach, particularly for high-dimensional search spaces.

References

- [1] Campana E. F., Liuzzi G., Lucidi S., Peri D., Piccialli V., Pinto A. (2009) New global optimization methods for ship design problems. *Optimization and Engineering*, vol. 10, pp. 533–555.
- [2] Tian M., Chen M., Du W., Tang Y., Jin Y., Yen, G. G. (2024) A composite decomposition method for large-scale global optimization. *IEEE Transactions on Artificial Intelligence*, vol. 5, pp. 6304–6319.
- [3] Yilmaz N. (2025) A new global optimization algorithm based on space-filling curve and auxiliary function approach and its applications. *Communications in Nonlinear Science and Numerical Simulation*, vol. 149, article 108920 (17 pages).

Simplicial and Rectangular Partitioning in Single- and Multi-Objective Global Optimization

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Keywords. Global optimization; non-convex multi-objective optimization; branch and bound; simplicial partitioning.

In this talk we discuss deterministic algorithms for global optimization, where many local minimizers may exist and the objective functions may be computable as black boxes. A general assumption is used that the objective functions satisfy the Lipschitz condition. We overview the branch and bound scheme for single- and multi-objective global optimization as well as bounds and bounding fronts based on the Lipschitz condition. We discuss various shapes of subregions including simplices and hyper rectangles, their advantages and disadvantages. We also investigate and compare bisection and trisection of subregions and various sampling strategies for evaluation of the objective functions in the subregions.

Spatial Prediction of Non-Gaussian Fields via a Data-Driven Stochastic PDE Approach

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Keywords. Spatial prediction; stochastic partial differential equations; non-Gaussian data.

Spatial prediction of continuous fields from incomplete observations is a central problem in many areas of computational science, including geostatistics, environmental modeling, and materials science [1]. Classical interpolation methods such as kriging typically rely on Gaussian assumptions, which can lead to suboptimal performance when applied to data with non-Gaussian (e.g., skewed or bimodal) distributions. Alternative approaches, such as indicator kriging or kernel-based regression methods, can handle non-Gaussian features but often involve increased computational complexity or require additional modeling choices.

We propose a novel numerical framework for spatial prediction based on a stochastic partial differential equation (SPDE) with a data-driven nonlinear potential. The method consists of three main components. First, the empirical probability density function of the observed data is estimated using kernel density estimation [2]. Second, this distribution is transformed into an effective potential function, which governs the local behavior of the field. Third, spatial correlations are introduced through a diffusion-like coupling term, and the resulting SPDE is solved numerically using a semi-implicit spectral scheme. The proposed approach can be interpreted as a stochastic relaxation procedure that evolves an initial random field toward a stationary state consistent with both the empirical distribution of the data and their spatial structure. Observed values are incorporated through hard or soft constraints applied during the evolution. The numerical solver is based on a Fourier-domain treatment of the linear operator, leading to efficient updates with computational complexity scaling approximately as $O(N \log N)$ per iteration for a grid with N nodes. In practice, convergence to a stationary regime is achieved in a relatively small number of iterations, resulting in near-linear scaling with problem size.

Acknowledgements.

This research was supported by the grant VEGA 1/0695/23.

References

- [1] Cressie N. (1993) *Statistics for Spatial Data*. Wiley, New York.
- [2] Silverman B.W. (1986) *Density Estimation for Statistics and Data Analysis*. Chapman & Hall, London.

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