Nonlinear Optimization, Machine Learning and Grossone

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The introduction of the grossone methodology, developed by Sergeyev, provides a novel computational framework to deal numerically with infinite and infinitesimal quantities within a unified positional numeral system based on the infinite unit ① (grossone) [4]–[6]. This approach has been successfully applied to optimization theory, allowing the construction of exact differentiable penalty functions for constrained nonlinear programs [3]. Penalty parameters traditionally required to tend to $+\infty$ can be replaced by suitable powers of grossone, and the finite part of the resulting solution recovers the optimal solution of the original constrained problem, while infinitesimal terms encode Lagrange multipliers.

Nonlinear optimization plays a central role in modern Machine Learning (ML), where most learning tasks are formulated as the minimization of empirical risk functionals possibly enriched with regularization terms. In this context, grossone has been employed to approximate and formulate important problems such as the ℓ_0 pseudo-norm through smooth expressions involving infinitesimals [2], the construction of spherical, linear and conical separators [7, 8].

Overall, the interplay between nonlinear optimization, ML models, and the grossone methodology opens promising perspectives: it provides exact penalty constructions, smooth approximations of combinatorial terms, and a unified numerical treatment of infinite parameters. These developments suggest that grossone-based techniques may constitute a powerful tool for large-scale learning problems where sparsity, constraints, and nonlinear structures coexist.

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