Spectral solution in time of evolutionary problems

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The numerical solution of Hamiltonian and, more generally, conservative problems has led to the definition of the class of energy-conserving Runge-Kutta methods named *Hamiltonian Boundary Value Methods (HBVMs)* [1,2]. Such methods, in turn, can be regarded as a truncated expansion of the vector field along the Legendre polynomial basis.

This interpretation of the methods has naturally led to their use as *spectral methods in time*, able to obtain spectral accuracy for general ODE problems. This has been possible by virtue of an extremely efficient Newton-type procedure for solving the generated discrete problems.

In so doing, such methods can be also used for efficiently solving multi-frequency, highly-oscillatory problems, with the possibility of using relatively huge time-step in the numerical solution.

Multi-frequency, highly-oscillatory problems naturally arise when using the methods of lines for solving Hamiltonian (or in general, hyperbolic) PDEs with the method of lines. Coupling a spectral space semi-discretization with a spectral accuracy in time, then allows solving very accurately this kind of problems with a highly efficient algorithm [3].

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[2] L. Brugnano, F. Iavernaro. Line Integral Solution of Differential Problems. *Axioms* 2018, 7(2), 36; https://doi.org/10.3390/axioms7020036

[3] L. Brugnano, F. Iavernaro, J.I. Montijano, L. Rández. Spectrally accurate space-time solution of Hamiltonian PDEs. *Numerical Algorithms* 81 (2019) 1183–1202. https://doi.org/10.1007/s11075-018-0586-z

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