Generalizations of the intermediate value theorem for approximations of fixed points and zeroes of continuous functions

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Generalizations of the tractional intermediate value theorem are presented. The obtained generalized theorems are particular useful for the existence of solutions of systems of nonlinear equations in several variables as well as for the existence of fixed points of functions and the localization and computation of extrema of objective functions.

The only computable information required by the numerical methods based on these generalized theorems is the algebraic sign of the function that is the smallest amount of information (one bit of information) necessary for the purpose needed, and not any additional information. Thus, these numerical methods are of major importance for tackling problems with imprecise (not exactly known) information. This kind of problems occurs in various scientific fields including mathematics, economics, engineering, computer science, biomedical informatics, medicine and bioengineering among others. This is so, because, in a large variety of applications, precise function values are either impossible or time consuming and computationally expensive to obtain.

Furthermore, these methods are particularly useful for tackling various problems where the corresponding functions take *very large* and/or *very small* values.

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