Grossone-based Infinity Computing with Numerical Infinities and Infinitesimals

Yaroslav D. Sergeyev

University of Calabria, Via Pietro Bucci, 42C, 87036 Rende (CS), Italy
Lobachevsky State University, Gagarin Av. 23, 603950 Nizhni Novgorod, Russia
yaro@dimes.unical.it

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In this tutorial, a recently developed computational methodology is described (see patents [1] and a comprehensive description given in [2]). It allows one to execute *numerical* (not symbolic) computations with a variety of infinities and infinitesimals based on the principle *'The part is less than the whole'* applied to all quantities: finite, infinite, and infinitesimal. This methodology does not contradict Cantor, Levi-Civita, and Robinson evolving their ideas in a more applied way and moving from purely symbolic to numerical calculus. A computational device called the *Infinity Computer* (see patents [1]) is used to work numerically with infinite and infinitesimal numbers that can be written in a positional system with an infinite base that is denoted by the symbol ① called *Grossone*. A number written in this numeral system can have several infinite and infinitesimal parts, a finite part can be either present or absent. On a series of theoretical and practical examples dealing with Turing machines, set theory, numerical differentiation, optimization, divergent series, ODEs, fractals, etc. (see, e.g., [2, 3, 4]) it is shown that the new way of counting can be very useful. Obtained results and their accuracy are repeatedly compared with answers provided by traditional tools used by mathematicians to work with objects involving infinity.

The *Infinity Calculator* working with the introduced infinities and infinitesimals numerically is shown during the presentation. A lot of information can be found at the dedicated web page http://www.theinfinitycomputer.com

References

[1] Ya. D. Sergeyev. Computer system for storing infinite, infinitesimal, and finite quantities and executing arithmetical operations with them: EU patent 1728149; RF Patent 2395111; USA patent 7,860,914.

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[4] Ya. D. Sergeyev, M. S. Mukhametzhanov, F. Mazzia, F. Iavernaro, and P. Amodio (2016) Numerical Methods for Solving Initial Value Problems on the Infinity Computer. *International Journal of Unconventional Computing*, Vol. 12(1), 3-23.