Numerical differentiation on the Infinity Computer and applications for solving ODEs and approximating functions

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The computation of derivatives using the Infinity Computer arithmetic has recently motivated the analysis of algorithms for the numerical solution of ordinary differential equations that involve higher derivatives. Moreover the relation of some of these algorithms with spline quasi-interpolation methods of differential type allowed us to use the infinite computer arithmetic also in the context of approximation theory.

After a review of the most important results already reached in this context, the talk will deal with Hermite-Obrechkov methods for the solution of ODEs and their related spline quasi-interpolation schemes. We shall see that the use of ① produces a precise value of the total derivative without explicitly evaluating its analytical expression in terms of the derivatives of f. The final goal of this new approach is to improve the computational effort associated with the evaluation of the involved derivatives and make them competitive with respect to more standard integrators. A novel application that will be described is the use of the Infinity Computer also in the approximation context, using ① to compute the input data of quasi-interpolation schemes.

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