Infinite Games on Finite Graphs using Grossone

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Finite board games that are played to infinity may sound like computer science fiction. Indeed, following the traditional Turing machine model, a computation is complete when it halts and produces some type of result. However when a game is played to infinity, it is implied that the game continues for an indefinite period (play continues without bound). It is easily seen that in a multiprogramming computer, when one process is satisfied, the operating system must continue and select another process to be executed. This is the ongoing operation and hence computation actually becomes a continuation action and, theoretically, without end. In the seminal work by Robert McNaughton (see [1]) a model of infinite games played on finite graphs is developed. The model proposes two players, Red and Black, that take turns moving from vertex to vertex along the directed edges of a bipartite graph, where each move corresponds to a different game configuration. This paper presents a new model of infinite games played on finite graphs using the Grossone paradigm (see [2] and [3]). The new Grossone model provides certain advantages such as allowing for a draw, which can result and are not uncommon in board games, and a more clear and decisive (accurate) method for determining the winner. Then, and subsequently, motivated by this infinite duration game model, a model to represent communication networks problems is proposed. Such a game model will be called an update network.

References

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