Emerging Utility

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Utility function seems to be the most disputable concept in the realm of mathematical economy. Analysis of prices and purchases on commodity markets add to this discussion the next paradoxical result: on one hand, behaviour of individual or household which has will and reflection CANNOT be described as maximization of any utility function, one the other hand, behaviour of large group of individuals (say, customers of one shop) which has no common will and reflection often CAN be described as maximization of some utility function.

To illustrate this thesis we consider simplified model of commodity market where we deliberately deprive consumers of information and options, which they usually have. Assume that there is the only trader in the market, which has stocks of different commodities. Customers come at random moments and ask to sell a certain bundle of commodities for certain amount of money. Stocks of trader are replenished with continuous steady flows. The trader knowing the distribution of orders and current stocks accepts or rejects the next order so as to maximize the expected discounted profit.

The optimal behaviour of the trader generates ergodic Markov process of changes in stocks. We investigate it by asymptotic expansion of the Bellman equation by small parameter ε which is the ratio of discounted factor to mean frequency of orders.

Consider what we will observe in our model market when replenish velocity $\mathbf{v}$ changes quasi-stationary. Stocks oscillate randomly in the vicinity of relative width $\varepsilon^{\frac{2}{3}}$ around equilibrium level $\bar{\mathbf{Q}}$ which connected with $\mathbf{v}$ by asymptotic relation $\bar{\mathbf{Q}} \sim (\mathbf{v}/\varepsilon)^{\frac{2}{3}}$. So the typical size of stocks is relatively large and depends on flow by the low often used in the inventory control theory. The trader selects proposals by cutoff price $\mathbf{p}(\mathbf{Q})$. In the typical vicinity of stocks the cutoff price depends on stocks unexpectedly week. $\mathbf{p}(\mathbf{Q}) = \bar{\mathbf{p}} + O(\varepsilon^{\frac{2}{3}})$. This means that the model explains usual economic pre-proposition on linear dependence of value on volume.

Collecting the trade statistics of average sales and equilibrium prices at different $\mathbf{v}$ we will see that this statistics may be rationalized by utility function $U(\mathbf{z}) = \min\limits_{\mathbf{p}} \left\{ \mathbf{p} \cdot \mathbf{z} + \int\limits_0^\infty d\mathbf{w} \int\limits_{\mathbf{p}\cdot\mathbf{w}}^\infty dW\, h(W, \mathbf{w})\ (W - \mathbf{p} \cdot \mathbf{w}) \right\}$, where $h(W, \mathbf{w})$ is the probability that the next customer will try to buy a bundle $\mathbf{w}$ for money W.