

# Metamathematical Investigations on the Theory of Grossone

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We will present Sergeyev's Grossone Theory as described in [1] or [2] as a formal system, stating its axioms for the natural numbers in the predicative second order language. We will discuss the possibility of interpreting his additional postulates as metamathematical constraint.

The formal theory is shown to be consistent by an application of the logical compactness theorem, without any reference to non standard analysis. It is also a conservative extension with respect to arithmetic, but this is not relevant, since the interesting results involve the use of the new constant  $\textcircled{1}$ .

Since  $\textcircled{1}$  is meant in particular to measure the size of sets of natural numbers, the axiomatization of its theory cannot be restricted to a first order language. Predicative second order logic should however suffice, and its logical strength is not greater than first order, only the expressive power.

We will address open problems concerning definability (of sets) in this theory.

We will consider in subsequent work Grossone theory as a theory of real numbers, but will try already to indicate how the main results extend to the larger setting.

## References

- [1] Sergeyev Ya. D. (2003) *Arithmetic of Infinity*. Edizioni Orizzonti Meridionali, CS.
- [2] Sergeyev Ya. D. (2008) A new applied approach for executing computations with infinite and infinitesimal quantities. *Informatica*, Vol. 19, No. 4, pp. 567–596.