Parametric Global Optimization: Application to Sailboat Robotics

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Introduction

A robot can generally be described by a vector first-order differential equation, named state equations. A robot is said to be redundant if it has more actuators than necessary. In this case, the number of inputs is higher than the number of outputs (variables to be controlled) and there exists many different ways to achieve the control requirements. We can thus take advantage of the extra number of freedom degrees in order to optimize some performance criterion (involving energy, security, longevity or speed). The resulting problem can be formalized into an parametric optimization problem with equality constraints where the free variables (or the parameters) of the optimization problem correspond to the outputs. Due the non-convexity of the optimization problem, the paper proposes to use an interval approach for the resolution. The approach is illustrated on the optimal sail tuning of a sailboat robot.

Formalism

Consider a mobile robot described by the following state equations

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{y} = \mathbf{g}(\mathbf{x}) \end{cases} \tag{1}$$

where $\mathbf{u} \in \mathbb{R}^m$ is the vector of inputs (or actuators) and $\mathbf{x} \in \mathbb{R}^n$ the state vector. The vector $\mathbf{y} \in \mathbb{R}^p$ is the vector of variables we want to control accurately. If $m > p$ the robot is overactuated and we will have different way to solve the control problem. In such a case, we may take advantage of this redundancies by maximizing a performance criterion $h(\mathbf{x})$. This criterion may correspond to the power delivered by the batteries (that we want to minimize) or to the speed of a boat (to be maximized), ... The corresponding optimization problem we have to solve is defined by

$$\hat{h}(\bar{\mathbf{y}}) = \max_{\bar{\mathbf{u}} \in \mathbb{R}^m, \bar{\mathbf{x}} \in \mathbb{R}^n} h(\bar{\mathbf{x}}) \qquad \text{s.t. } \begin{cases} \mathbf{0} = \mathbf{f}(\bar{\mathbf{x}}, \bar{\mathbf{u}}) \\ \bar{\mathbf{y}} = \mathbf{g}(\bar{\mathbf{x}}). \end{cases} \tag{2}$$